Peurbach’s *Theoricae novae planetarum*

A Translation with Commentary

*By E. J. Aiton*

GEORG PEURBACH (Georgius Aunpekh de Peurbach) was born in the Austrian town of Peuerbach, a few miles west of Linz. Although his date of birth is generally given as 30 May 1423—on the basis, apparently, of a horoscope published in 1550—the reliable evidence only indicates some date after 1421.¹ Peurbach received his bachelor’s degree in the Arts Faculty of the University of Vienna in 1448, having probably already begun his study of astronomy under followers of John of Gmunden. He then traveled through France, Germany, and Italy, where it is likely that he met Giovanni Bianchini, the most noted Italian astronomer of the time. On his return to Vienna in 1453, he received the master’s degree, and through his university lectures on Roman poets he played a leading role in the revival of classical learning that had been initiated by Aeneas Silvius Piccolomini. Following the advice of Johann Nihil, the court astrologer to Emperor Friedrich III in Wiener-Neustadt, Peurbach accepted the position of court astrologer to the king of Hungary and sometime later himself became imperial astrologer.

In 1460, at the request of Johannes Bessarion, archbishop of Niceae, Peurbach undertook the preparation of an abridgment of Ptolemy’s *Almagest*, known to the astronomers of the fifteenth century in the Latin translation from the Arabic of Gerard of Cremona made at the end of the twelfth century.² There was also a Latin translation from the Greek by George of Trebizond, but this was considered to be unsatisfactory. Peurbach completed only the first six books of his

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**Figure 1.** The opening page of Peurbach's Theoricæ novæ planetarum (Venice, 1485), folio 3.2v. Courtesy of the Wellcome Institute Library, London.
abridgment before he became mortally ill; shortly before his death on 4 April 1461 he obtained from his student Regiomontanus a promise to complete the work, which the latter did in Italy during the next year or two. The *Epytoma Joannis de Monte Regio in Almagestum Ptolemei* was first printed in Venice in 1496, twenty years after the death of Regiomontanus.\(^3\) It not only provided a relatively simple account of Ptolemy’s complex masterpiece but also contributed to current research by adding later observations, revised computations, and critical reflections, especially concerning the failure of Ptolemy’s lunar theory to represent the appearances.

At the time that he embarked on the abridgment of the *Almagest*, Peurbach was already well known for his *Theoricae novae planetarum*, an astronomical textbook designed to replace the popular *Theorica planetarum* generally ascribed to Gerard of Cremona.\(^4\) For over two hundred years the *Theorica planetarum*, together with the *Sphere* of Sacrobosco (an introduction to spherical astronomy and astronomical geography), had provided students with the preliminary knowledge needed for study of the *Canones* to the astronomical Alfonso’s Tables, astronomical treatises such as those of al-Battānī and al-Farghānī, and the *Almagest* itself. The superiority of Peurbach’s excellent work soon became evident and was firmly established following the publication of Regiomontanus’s famous *Disputationes contra deliramenta Cremonensis*.\(^5\)

The *Theoricae novae planetarum* originated as a series of lectures given by Peurbach in 1454 in the Collegio Civium (Bürgerschule) in Vienna. Among the audience was Regiomontanus. His copy of the work, completed in his own hand on 30 August, is one of the earliest surviving manuscripts.\(^6\) A section on Thābit ibn Qurra’s theory of trepidation was added in about 1460, as a reference in the printed editions to this year as being “in our time” would appear to indicate—probably by Peurbach, for Regiomontanus is unlikely to have added a section of his own during Peurbach’s lifetime. (If Regiomontanus had composed it, he would probably have done so for the printed edition of 1472, and is then unlikely to have chosen the year 1460 to represent “our time.”) The problem of the authorship of this addition is complicated by the facts that an early manuscript version of it evidently does not exist and that Peurbach’s original manuscript, of which that of Regiomontanus is a copy, is lost.\(^7\)

Regiomontanus published the first printed edition in Nuremberg in 1472. This

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\(^4\) Olaf Pedersen, “The *Theorica planetarum* Literature of the Middle Ages,” *Classica et Mediaevalia*, 1962, 23:225-232, on pp. 230-231, discusses the authorship, pointing out that manuscripts earlier than the thirteenth century are not known. However, Richard Lemay has found a twelfth-century manuscript formally ascribed to Gerard of Cremona; see Lemay, “Gerard of Cremona,” in *Dictionary of Scientific Biography* (cit. n. 1), Vol. XV, pp. 173-192, on p. 189. This seems to suggest that Gerard was most probably the author. There is a modern critical edition, *Theorica planetarum* *Geraldii*, ed. Francis J. Carmody (Berkeley, Calif.: privately published, 1942).


\(^6\) Österreichische Nationalbibliothek, Vienna, Codex 5203, fols. 2r-24r. At the end of the manuscript Regiomontanus wrote: “finiunt Theoricae novae per magistrum Georgium de peurbach edite anno domini 1454 Wienne in Collegio civium penultima mensis Augusti.”

PEURBACH'S THEORICAE NOVAE PLANETARUM edition, which included some beautiful colored diagrams, follows the text of Regiomontanus's manuscript, apart from the addition of the section on Thabit ibn Qurra's theory of trepidation, trivial changes of word order, minor corrections, the addition of a diagram illustrating the aspects, and the omission of five lines in one place, which is indicated in the translation. Over the next century many further editions appeared, some with commentaries. While the commentators left the text unchanged, they often amended Peurbach's diagrams in the interests of clarity.

Ernst Zinner records the existence of nine manuscript copies and fourteen manuscript commentaries. This certainly underestimates the total number, for a recent survey has revealed the existence of twelve manuscript texts or commentaries in Cracow alone. Among printed commentaries are those of Albert of Brudzewo (1495), Joannes Capuan (1495, 1499, 1503, 1508, 1518), Erasmus Reinhold (1542, 1553, 1555, 1558, 1562, 1580, 1601, 1604, 1653), Oswald Schreckenfuchs (1556), and Pedro Nunes Salaciense (1566, 1573). In 1566 Oratio Toscanella published an Italian translation in Venice, and a Hebrew translation dating from 1546 exists in manuscript. Peurbach's theories were described in French by Oronce Fine in his La théorique des ciels, mouvements et termes practiques des sept planetes, first published in Paris in 1528 and again in 1557, 1607, and 1619. Altogether at least fifty-six printed editions, including translations and commentaries, appeared between 1472 and 1653. Peurbach's work was the favorite astronomical textbook of the sixteenth century, and its influence extended beyond the time of Kepler almost to the beginning of the Newtonian era.

Peurbach's Theoricae novae planetarum introduces the reader to the Ptolemaic geometrical models as embodied in the physical realizations described by Ptolemy in his Planetary Hypotheses. Peurbach evidently drew upon Ibn al-

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9 Ernst Zinner, Verzeichnis der astronomischen Handschriften der deutschen Kulturgebiete (Munich, 1925), nos. 7691-7714; and Emmanuel Poule, "Astronomy in Cracow," Journal for the History of Astronomy, 1985, 16:134-138, on p. 137. Some of the manuscript texts also contain marginal annotations; e.g., Regiomontanus's manuscript contains annotations by the astrologer Johannes Schöner (1477-1547). Some of the later manuscripts include the section on Thabit ibn Qurra's theory of trepidation; e.g., Bayerische Staatsbibliothek, Munich, CLM (= Codex latinus Monacensis) 51, fols. 72-188, and CLM 22639, fols. 75-118. Both of these manuscripts lack the lines mentioned in note 113 to the translation but include Figure 18 on the aspects (which does not appear in the manuscript copied by Regiomontanus) and also the colored diagrams as they appear in the early printed editions. From these facts it is possible to infer that the manuscripts were probably copied from an early printed edition. The first is a beautifully written copy. The second contains annotations but lacks Figure 28 on the "Theorica ad terminos spectans." On the relations between the manuscripts see Grossing, Humanistische Naturwissenschaft (cit. n. 1), pp. 135-139.
10 Oratio Toscanella, Le nuove teoriche de i pianeti di Georgio Purbachio (Venice, 1566); the Hebrew manuscript is in the Biblioteca Mediceo-Laurenziana, Florence (fondo Ashburnham). See also Houzeau and Lancaster, Bibliographie (cit. n. 8), Vol. I, Pt. 1, p. 551; and Zinner, Leben und Wirken (cit. n. 1), p. 34.
11 Although this work is described in the bibliographies as a translation of the Theoricae novae planetarum, it is in fact a very free adaptation of part of Peurbach's text; Peurbach's name is nowhere mentioned. The author claims to be following the tradition of the modern and more approved astronomers.
12 This work consists of two books. Only the first part of Book I is extant in Greek, but the whole work survives in Arabic. The Greek text together with a German translation of both it and the Arabic version of Book II have been published in Claudii Ptolemaei opera quae exstant omnia, ed. J. L. Heiberg (Leipzig: Teubner, 1907), Vol. II. An English translation of the remaining part of Book I,
Haytham’s (Alhazen’s) *On the Configuration of the World* or some later work based on this.\(^{13}\) Although it is clear from the incipit of the manuscript version that Peurbach intended his systems of complementary spheres and spherical shells to be regarded as real and not simply as geometrical illustrations for teaching purposes, the title chosen by Regiomontanus for the printed edition hides this fact.\(^{14}\) There is no doubt that both Ptolemy and Alhazen regarded such devices as real, and Albert of Brudzewo, in one of the earliest commentaries on Peurbach’s work, which was used in lecture courses at Cracow when Copernicus was a student there, described Peurbach’s spheres as real and his circles as imaginary.\(^{15}\) Following the sections on the sun, moon, and planets, Peurbach gave an account of the motion of the eighth sphere, describing the theories of precession of al-Battâni and al-Farghani, which had formed the basis of the Alfonsine Tables. This completed the work in its first form. When, a few years later, either Peurbach or possibly Regiomontanus added the description of the theory of trepidation by means of which Thâbit ibn Qurra explained the precession, no change was made in the original account of the motion of the eighth sphere.

Peurbach supported Ptolemy’s geocentric model of the universe. Although he does not mention the possibility of the earth’s motion in the *Theoricae novae planetarum*, elsewhere he twice relates Ptolemy’s arguments against the daily rotation of the earth, supposed by “certain people”: in the part of the *Epitome* he himself composed and in a scholastic disputation entitled “An terra moveatur an quiescat.”\(^{16}\) In another work he once mentions Aristarchus of Samos, but he nowhere discusses the possibility of a heliocentric system.\(^{17}\) There is, however, a

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\(^{17}\) For the reference to Aristarchus, see Georg Peurbach, “Positio sive determinatio de arte oratoria sive poética,” in *Die Frühzeit des Humanismus und der Renaissance in Deutschland*, ed. Hans Rupprich (Leipzig: Reclam, 1938), pp. 197–210, on p. 205. Grössing, *Humanistische Naturwissenschaft* (cit. n. 1), p. 91, is mistaken in supposing that Peurbach could have found a discussion of heliocentrism in Ptolemy’s *Almagest*. Grössing also suggests that Cusanus provides another source for inspiration concerning the possibility of the earth’s rotation on its axis (p. 94). On the interpretation of Cusanus, however, see Aiton, “Celestial Spheres” (cit. n. 14), p. 92.
statement in the *Theoricae novae planetarum* that is highly suggestive of a primary role for the sun in the motions of the planets. Peurbach writes, "It is evident that the six planets share something with the sun in their motions, and that the sun is like some common mirror and rule of measurement to their motions." Such a statement could inspire equally well the formulation of the heliocentric system of Copernicus or the heliocentrism of importance that Ficino derived from the Hermetic tradition.18

Two other items probably have some connection with the lectures that formed the *Theoricae novae planetarum*, both relating to presentation of the theories. Peurbach composed a short manuscript, "Speculum planetarum," in which he described the construction of paper equatoria with revolving discs and threads to represent the motions of the planets on the basis of the Alfonsine Tables.19 Also in the Stadtmuseum in Schweinfurt, near Nuremberg, there exists today a small planetarium that may have been constructed to illustrate the theory of trepidation explained in the lectures.20

The following translation of Peurbach’s *Theoricae novae planetarum* is based on the Ratdolt edition published in Venice in 1485. This edition is bound up with the *Sphere* of Sacrobosco and the *Disputationes contra deliramenta Cremonensis* of Regiomontanus.21 The diagrams have been reproduced photographically from this edition, apart from three that have been redrawn.

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19 Vienna Codex 5203, fols. 88–92.
21 There is a facsimile reproduction of an earlier edition in Regiomontanus, *Opera*, pp. 755–793. This is described as having been published in Venice in about 1472, but it is in fact the edition published in Nuremberg at about that time. The facsimile is incomplete, the last page having been omitted. Numbers in brackets in the margins of the translation below refer to the signature and folios of the 1485 edition.

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NEW THEORIES OF THE PLANETS

*By Georg Peurbach*

ON THE SUN

The sun has three orbs, separated from one another on all sides and also contiguous [3.2v] to one another [Figure 1]. The highest of them is concentric with the world on its convex surface, but is eccentric on its concave surface. The lowest, on the other hand, is concentric on its concave but eccentric on its convex surface. The third, however, situated in the middle of these, is eccentric to the world on both its convex surface and its concave surface. Now an orb whose center is the center of the world.

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Figs. 1, 4, 7, 8, 9, 12, and 16 are reproduced by courtesy of the Wellcome Institute Library, London. Figs. 2, 3, 5, 6, 10, 11, 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, and 29 are reproduced by courtesy of the British Library, London.

I am indebted to the Österreichische Nationalbibliothek, Vienna, and the Bayerische Staatsbibliothek, Munich, for supplying microfilms of manuscripts of Peurbach’s *Theoricae novae planetarum* in their possession. Finally, I should like to thank the anonymous referee who sent me a copy of his unpublished translation, which I have used to improve my own translation of the section on eclipses.

1 Cf. the illustration in Ptolemy’s *Planetary Hypotheses* (Claudii Ptolemaei opera quae exstant omnia, ed. J. L. Heiberg [Leipzig: Teubner 1907], Vol. II, p. 132). The word *orbis*, when meaning, as here, a spherical shell of either uniform or nonuniform thickness, is translated as “orb” in order to distinguish it from *sphaera*.
[3.3r] is said to be concentric with the world, and an orb whose center is other than the center of the world to be eccentric. Therefore the first two are eccentric relatively, and they are called the deferent orbs of the apogee of the sun. The apogee of the sun varies according to their motion. The third is eccentric absolutely and is called the deferent orb of the sun. The body of the sun is attached to it and moves indeed according to its motion. These three orbs take two centers. For the convex surface of the highest and the concave of the lowest have the same center, which is the center of the world. From that fact the whole sphere of the sun, just as the whole sphere of any other planet, is said to be concentric with the world. But the concave surface of the highest orb and the convex of the lowest, together with the surfaces of each side of the middle orb, share another center, which is called the center of the eccentric.

[3.3v] The deferent orbs of the apogee of the sun move by their own proportional motions, so that the narrower part of the superior is always above the wider part of the inferior, and go around equally fast, following the variations of the motion of the eighth sphere, concerning which we shall speak later. Nevertheless the poles of this motion are those of the ecliptic of the eighth sphere [Figure 29]. For the apogee of the eccentric deferent of the sun continually revolves in the plane of this ecliptic. But the deferent orb of the body of the sun, by its own motion on its center, that is, the center of the eccentric, moves each day, uniformly eastward, fifty-nine minutes and about eight seconds in relation to the circumference described in one complete revolution by the center of the body of the sun. The poles of this motion are distant from the poles of the previous orbs and are termini of the axis of this former [eccentric] orb, namely, the line drawn through the center of the eccentric, parallel to the axis of the deferent orbs of the apogee. From these facts it appears that, through the motion of the deferent orbs of the apogee, which they have by virtue of the motion of the eighth sphere, the axis of the solar deferent orb with the center of the eccentric circle, and also the poles of the solar deferent orb, describe the circumferences of small circles about the axis of the deferent orbs of the apogee, according to the size of their eccentricity. Since the center of the sun itself moves uniformly about the center of the eccentric in accordance with the motion of the deferent orb, it follows that it must move nonuniformly about any other point. For this reason the

2 orbis augem solis deferentes. The term aux (gen. augis) is the medieval Latin equivalent of awj, the term introduced by the Arabs to describe the apogee. Peurbach uses the standard medieval terms, which were also used in the Theorica planetarum of Gerard of Cremona. See Olaf Pedersen, “A Fifteenth Century Glossary of Astronomical Technical Terms,” Classica et Mediaevalia, 1973, 34:584–594 (hereafter Pedersen, “Glossary”).

3 Regarding the ecliptic and equinoxes as fixed, Ptolemy took the tropical year—the interval between the sun’s successive returns to the same equinox—as the fundamental astronomical concept in the case of the motion of the sun. He considered precession, discovered by Hipparchus, to be a slow eastward motion of the sphere of the fixed stars—the eighth sphere—about an axis through the poles of the ecliptic. Ptolemy supposed the solar apogee to be fixed in longitude: Ptolemy’s Almagest 3.4, ed. and trans. G. J. Toomer (London: Duckworth, 1984), p. 153 (hereafter Almagest). Besides sharing in the motion of the eighth sphere (precession), as Peurbach notes here, the solar apogee has an additional eastward motion of about 12° per year. This motion was first clearly distinguished from precession by al-Birūnī; see W. Hartner and M. Schramm, “Al-Bīrūnī and the Theory of the Solar Apogee,” in Scientific Change, ed. A. C. Crombie (London: Heinemann, 1963). It was assigned a definite value by al-Zarqāl in about 1050; see G. J. Toomer, “The Solar Theory of al-Zarqāl,” Centaurus, 1969, 14:306—336, on p. 316.

4 orbs solem deferens.

5 tota sphaera. The whole spheres of the planets (containing the required eccentric and epicyclic orbs or spheres within them) were the contiguous homocentric spherical shells described by Ptolemy in the Planetary Hypotheses; see Bernard R. Goldstein, “The Arabic Version of Ptolemy’s Planetary Hypotheses,” Transactions of the American Philosophical Society, N.S., 1967, 57(4):1–55, on p. 8.

6 In the diagram the labels c mundi and c deferentis are displaced downward from their proper positions. It should be noted that the orbs rotate out of the plane of the paper. The semicircles (considered to be in parallel planes perpendicular to the paper) represent the motion of the axis of the solar deferent about the axis of the ecliptic of the eighth sphere.

7 secundum successionem signorum.

8 axi orbium augem deferentium aequidistantis.
sun in equal times describes unequal angles about the center of the world and unequal arcs on the circumference of the zodiac. Therefore the eccentric circle is called the circle of the point of egress or of the outward center, whose center is other than the center of the world, although encircling this.

We imagine then in the case of the sun an eccentric circle to be described, with uniform motion about the center of the eccentric, by a line drawn from the center of the eccentric up to the center of the sun making one revolution. This circle is always part of the plane of the ecliptic of the orb of signs of the eighth sphere.

The apogee of the sun [Figure 3] in its primary meaning is the point of the circumference of the eccentric most distant from the center of the world. Its position is determined by the line drawn on both sides from the center of the world through the center of the eccentric, which is called the line of the apogee.

The perigee or the least distance is the point of the circumference of the eccentric nearest to the center of the world and always diametrically opposite the apogee.

The mean distance is a point of the circumference between the apogee and perigee. And its position in the case of the sun is determined by a line that, going from the center of the world, makes right angles with the line of the apogee. Only two such points are found in the same eccentric.

The line of mean longitude of the sun is the line extended from the center of the world to the zodiac, parallel to the line taken from the center of the eccentric to the center of the sun. Nevertheless these two lines coincide twice in a year, when the sun is in the apogee or in the perigee of the eccentric. And if one of the two lines revolves uniformly about its center, so also does the other about its center. For when they do not coincide, they both always make equal angles with the line of the apogee.

The mean longitude of the sun is the arc of the zodiac reckoned eastward from
the beginning of Aries up to the line of mean longitude. The apogee of the sun in its secondary meaning\(^\text{15}\) is the arc of the zodiac reckoned eastward from the beginning of Aries up to the line of the apogee.

The argument\(^\text{16}\) of the sun is the arc of the zodiac between the line of the apogee and the line of mean longitude of the sun, reckoned eastward. This arc is always similar to the arc of the eccentric falling eastward between the apogee of the eccentric and the center of the sun. For this reason it follows that, if the apogee of the sun in its secondary meaning is subtracted from the mean longitude of the sun, or from this with a whole circle added,\(^\text{17}\) there remains the argument of the sun.

The line of the true longitude of the sun is the line from the center of the world extended through the center of the body of the sun to the zodiac, which coincides with the line of mean longitude when the sun is in apogee or perigee.

The true longitude\(^\text{18}\) of the sun is the arc from the beginning of Aries up to the line of true longitude. The mean longitude and the true longitude are the same only when the sun is in apogee or perigee, for otherwise they are always different.

The equation\(^\text{19}\) of the sun is the arc of the zodiac falling between the lines of mean and true longitude. This becomes zero whenever the sun is in apogee or perigee. The greatest that it can be occurs when the sun is in the mean distances. But when the sun is in other places, it increases and decreases according to the variation of the argument. For when the sun is nearer the apogee or perigee, the equation is so much the less, and when nearer the mean distances, so much the more. When the argument is less than six signs in all, the line of mean longitude precedes the line of true longitude, and therefore the equation is then subtracted. But when it is more than six signs, the opposite is true, and therefore the equation is then added to the mean longitude in order to obtain the true longitude of the sun.

ON THE MOON

The moon has four orbs and one small sphere.\(^\text{20}\) First, it has in fact three orbs, like the sun, arranged as in the diagram [Figure 4]; namely two relatively eccentric orbs, which are called the deferent orbs of the apogee of the moon's eccentric, and a third, absolutely eccentric, placed in the middle of them, which is called the deferent of the epicycle. Next, it has an orb concentric with the world, which is attached to the other three and revolves round them; this is called the deferent of the head of the dragon. Last, it has a small sphere, which is called an epicycle, immersed in the depth of the third orb; in fact, the body of the moon is fixed to this epicycle.

Now the deferents of the apogee of the eccentric move westward together, uniformly about the center of the world, by about eleven degrees and twelve minutes beyond the diurnal motion in a natural day. And the axis of this motion intersects the axis of the zodiac in the center of the world, and from there its poles decline from the poles of the zodiac, and the measure of such declination is five degrees and always constant. The deferent orb of the epicycle moves eastward uniformly about the center of the world, so that in every natural day the center of the epicycle traverses by such a motion about thirteen degrees and eleven minutes. Nevertheless the axis of this motion through the center of this orb, which is called the center of the eccentric, moves parallel to the axis of the deferents of the apogee. As an addi-

\(^{15}\) aux solis in secunda significatione is therefore the ecliptic longitude of the solar apogee.

\(^{16}\) Because the term "anomaly" is used in different senses in the Almagest, the medieval astronomers used the term argumentum to describe the concept defined here. This will be translated as "argument." It relates the position of the sun to the line of apsides.

\(^{17}\) The addition of 360° is necessary when the line of mean longitude is west of the line of the apogee.

\(^{18}\) verus motus solis.

\(^{19}\) aequatio solis.

\(^{20}\) sphaerula. The term sphaera is used in two senses by Peurbach. The first, in the expression tota sphaera, is explained in n. 5 above. Here the term means a sphere with one surface, i.e., a solid sphere in the geometrical sense; see E. J. Aiton, "Celestial Spheres and Circles," History of Science, 1981, 19:75–114, on pp. 94–95.
tional consequence the poles of that motion will be distant from the poles of the
deferent orbs of the apogee, according to the size of the eccentricity [Figure 5].

From this it follows, first, that although the eccentric deferent of the epicycle [3.5v] moves about the axis and also its poles, it still does not move uniformly about them.22

Second, to the extent that the epicycle of the moon is closer to the apogee of the
deferent, its center moves so much the faster; and to the extent that it is closer to
the perigee of the same, so much the slower. For if any equal angles are marked about the center of the world toward the apogee and perigee, the angle that is
toward the apogee includes a greater arc of the eccentric than the other, which is
toward the perigee [Figure 623].

Third, the center of the eccentric of the moon revolves uniformly about the center
of the world, the axis of the same orb revolves uniformly about the axis of the
deferents of the apogee, and the poles of the same revolve uniformly about the poles
of those, all describing circumferences westward.24

Fourth, the apogee of the eccentric of the moon will move likewise westward with
uniform progression and will cross over the ecliptic. Hence the apogee is found
sometimes in the plane of the ecliptic, sometimes, however, away from this plane,
either toward the south or the north. Consequently, it happens that the center of the
eccentric also sometimes recedes in like manner from the plane of the ecliptic in
opposite directions.

Fifth, the plane of the ecliptic will not always cut the plane of the eccentric in

21 The lunar theory described here by Peurbach is Ptolemy's second lunar model; *Almagest* 5.2, pp. 220–222. It was designed to include the second lunar anomaly (later called the evection), the discovery of which was one of Ptolemy's outstanding contributions to astronomy; see Olaf Pedersen, *A Survey of the Almagest* (Odense: Odense Univ. Press, 1974), pp. 184–187 (hereafter Pedersen, *Survey*).

22 Here Ptolemy for the first time departed from the traditional principle of uniform circular motion. Although he always claimed his adherence to the principle, he nevertheless tacitly departed from it when he found it convenient to do so; Pedersen, *Survey*, p. 35.

23 In this diagram the outer circle represents the ecliptic.

24 *contra successionem signorum*. Thus the center of the eccentric deferent moves westward in a small circle about the center of the world. This circle was called the *circulus parvus* by the medieval astronomers. It was introduced by Ptolemy to draw the center of the epicycle nearer to the earth at the quadratures. Peurbach indicates this motion of the center of the eccentric and of its poles by the small semicircles in Fig. 5. The north and south poles are indicated by $S$ (septentriones) and $M$ (meridies), respectively.
equal parts. When in fact the apogee of the eccentric has a latitude, the greater part of the plane of the eccentric will be toward the apogee. For the plane of the eccentric is cut by the plane of the ecliptic in a diameter of the ecliptic passing through the center of the world.  

The circle described by a revolution of the line from the center of the eccentric extended to the center of the epicycle is called the plane of the eccentric. The parts of its circumference (as in the case of the sun) are called the apogee and perigee and also the mean distances. The motion of the three lunar orbs has such a relation to the motion of the sun that the line of mean longitude of the sun is always in the middle, between the center of the epicycle of the moon and the apogee of its eccentric, appearing either with them or in the opposite position when both of them are together. Thus in every mean conjunction of the sun and moon, the center of the epicycle of the moon, the line of mean longitude of the sun, and the apogee of the eccentric of the moon are in one point of the zodiac, according to longitude. For this reason it happens that in all their mean quadratures the center of the epicycle of the moon is in the perigee of its eccentric, and in every mean opposition again in the apogee.  

Consequently, the reason is plain why by subtracting the mean longitude of the sun from the mean longitude of the moon, the remainder is their mean elongation, and from twice this, the centrum of the moon is produced. For the distance eastward of the line of mean longitude of the moon from the line of mean longitude of the sun is called their mean elongation.  

Moreover, the distance eastward of the line of mean longitude of the moon from the apogee of the eccentric is called the centrum of the moon, or the double distance or double interval. Also it appears that in every lunar month, the center of the epicycle of the moon twice passes through the orbs that carry the apogee of the eccentric. But the fourth concentric orb, carrying the head of the dragon, moves on the axis of the zodiac, revolving westward uniformly about the center of the world, about three minutes in every natural day, together with such motion as is continually added from the three orbs that it surrounds. Because of this motion it happens that the circumference of the eccentric continually intersects the plane of the ecliptic toward the west at some point or other.  

Again it follows that by such motion the poles of the deferents of the apogee, by moving about the poles of the zodiac, describe the circumferences of circles. Now the epicycle, carrying the body of the moon, which is attached to it, about its center, westward in the upper part and eastward in the lower part, moves on its axis. The axis lies perpendicular to the circumference of the epicycle, so that the plane of the circumference of the epicycle, which the center of the moon describes by the motion of the epicycle, remains in the plane of the eccentric, never departing from that.  

The epicycle, however, revolves in such a way that it moves nonuniformly about its own center and on its own axis. But this irregularity is reduced to uniformity in that the moon is uniformly removed from the point of the mean apogee of the epicycle,  

25 Only when the apogee of the eccentric is in the nodes will the division be equal.  
26 Cf. the description in Pedersen, Survey, p. 187.  
27 The term *centrum lunae* for twice the elongation (i.e., the angular separation of the sun and moon) was introduced by the medieval astronomers because it measures the angular distance of the center of the moon's epicycle from the moving apogee of the eccentric deferent; Pedersen, "Glossary," p. 589.  
28 The terms "head" and "tail of the dragon" (*caput et cauda draconis*) refer to the ascending and descending nodes, respectively. They are derived from the mythological explanation of eclipses, found with variations in ancient India, China, and Islam, according to which a dragon with head and tail twisted around the nodes swallowed the sun and moon whenever the opportunity occurred.  
29 This is the difference between the mean daily motion in latitude and the mean daily motion in longitude, i.e., the difference between the mean daily motions calculated from the draconitic month and the tropical month; *Almagest* 4.3, p. 179; Pedersen, Survey, pp. 160-164.  
30 These are indicated by the large semicircles in Fig. 5.  
31 Thus in Ptolemy's lunar theory the epicycle and the eccentric are in the same plane, inclined at a constant angle to the plane of the ecliptic.
wherever that is, by receding in every natural day by thirteen degrees and about
four minutes.32

Now the mean apogee of the epicycle33 is the point of the circumference of
the epicycle, which is determined by the line drawn through the center of the epicycle
from the point diametrically opposite the center of the eccentric in the small circle.
But the true apogee of the epicycle is the point of the circumference that is deter-
mined by the line drawn through the center of the epicycle from the center of the
world. These two apogees coincide when the center of the epicycle is in the apogee
or perigee of the deferent; everywhere else, however, they diverge.34

From these considerations it appears that no identical point of the cavity in which
the epicycle is situated remains continuously over the mean or true apogee of the
epicycle. For such a point of the cavity, which, when the center of the epicycle is in
the apogee or perigee of the deferent, will be over the mean or true apogee of the
epicycle, is always determined (wherever the center of the epicycle is) by the line
drawn from the center of the eccentric through the center of the epicycle. If the
center of the epicycle is elsewhere than in the apogee or perigee, however, such a
point is over neither the mean apogee nor the true apogee. On the contrary, the true
apogee and the mean apogee are then under other places of the same cavity, for the
three lines that show the above mentioned points will then intersect in the center of
the epicycle. But this will be in such a way that the true apogee (when it differs from
the mean apogee) is always between the mean apogee and the point of the cavity
under which the true apogee usually is, when the center of the epicycle is in
the apogee or perigee of the deferent. For this reason it follows that both the mean and
true apogee of the epicycle are continually changed. It follows again from this that
the revolution of the epicycle about its own center is faster when the center of the
epicycle is traversing the upper half of the eccentric and slower when it is traversing
the lower half.

Therefore the line of mean longitude of the moon is that extended from the center
of the world to the zodiac through the center of the epicycle [Figure 6].

The mean longitude of the moon is the arc of the zodiac from the beginning
of Aries up to the said position. From what has been explained the centrum of the
moon is made clear.

The line of the true position or true longitude of the moon is that extended from
the center of the world, through the center of the body of the moon, up to the
zodiac.35

The true longitude of the moon is the arc of the zodiac from the beginning of Aries
up to the said line.

The equation of center is the arc of the epicycle intercepting its true apogee and
mean apogee. This becomes zero when the center of the epicycle is in the apogee or
perigee of the eccentric. It is greatest when this center is a little below the mean
distances of the deferent.

The mean argument of the moon is the arc of the epicycle, reckoned in the direc-
tion of the motion of the center of the body of the moon from the mean apogee of the
epicycle up to the same lunar center.

The true argument, however, extends from the true apogee up to the center of the
body of the moon. Therefore the difference between these arguments, when they

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32 The lunar model described so far was designed to give correct positions at the syzygies and the
quadrate. Discrepancies at the octants led Ptolemy to the refinement introduced here, whereby the
uniform rotation of the epicycle is referred not to the true apogee but to a different point, which he
called the mean apogee; *Almagest* 5.5, pp. 226–233; Pedersen, *Survey*, pp. 192–195. This theory of
the epicycle is peculiar to the moon.

33 Extracts from Peurbach's lunar theory (beginning here) are translated into German in Helmuth
extract, however, is out of sequence.

34 In Fig. 6, the difference of the mean and true apogee of the epicycle in the mean distances is
shown by the lines drawn through the center of the epicycle.

35 In Fig. 6, the lines of true longitude are those from the center of the world to the dots (represent-
ing the moon) on the epicycle.
differ, is the equation of center. When indeed the center of the epicycle of the moon is less than six signs, the true argument is greater than the mean; for that reason the equation of center is added to the mean argument. But when the center is more than six signs, the converse is the case, so that then the equation of center is subtracted in order to obtain the true argument.

The equation of the argument is the arc of the zodiac lying between the lines of mean and true longitude. This turns out to be zero when the center of the lunar body is in the true apogee or perigee of the epicycle, wherever the center of the epicycle then may be. The equation of the argument is, however, greatest when the center of the epicycle is in the perigee of the eccentric and when in addition the moon is in the line drawn from the center of the world at a tangent to the circumference of the epicycle. When however the true argument is less than six signs, the line of mean longitude precedes the line of true longitude, eastward; for that reason the equation of the argument is then subtracted from the mean longitude. But when the true argument is more than six signs, the converse is the case; so that then the equation of the argument is added in order to obtain the true longitude. Nevertheless the equations of the same arguments vary as the center of the epicycle goes from the apogee to the perigee of the deferent, for the equations continually increase according to the approach of the center of the epicycle to the center of the world. Consequently it happens that the equations of individual arguments that occur when the center of the epicycle is in the perigee of the eccentric are greater than the equations of the individual arguments that occur when the center of the epicycle is in the apogee of the eccentric, comparing equations to their corresponding arguments. Again the excesses of the former over the latter are called the variations of the diameter of the small circle. \[36\] In fact the line taken from the center of the world to the apogee of the deferent is longer than the line extended from the same center to the perigee. Moreover, the excess of the former over the latter, divided into sixty equal parts, is called proportional parts \[37\] and is twice the eccentricity [Figure 7].

For that reason, the line of mean longitude of the moon that is directed to the apogee of the eccentric holds none of these parts beyond the circumference of the eccentric, but all within; however, the line that reaches out to the perigee holds all beyond, none within. But those lines that are extended to other places of the same eccentric have some of these parts beyond—more, if the center of the epi-

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cycle is near to the perigee, and fewer in proportion, if it is near to the apogee.

Now the equations of the arguments that are written in the tables are those that come about when the center of the epicycle is in the apogee of the deferent. But these (as stated above) are less than those that are found when the center of the epicycle is in other places. When therefore the center of the epicycle is in other places (which happens when the centrum of the moon is greater than zero), the proportional parts in the tables are given through the centrum, and the variation of the diameter is given through the true argument, the total of which is added to the equation of the argument first taken from the tables if the proportional parts are sixty. But if they are less, the total is not added, but some portion of it, in the same ratio as are the proportional parts in relation to sixty; then the result will be the equation of the true argument for that position of the epicycle.

**ON THE DRAGON OF THE MOON**

The plane of the eccentric of the moon, as stated above, by reason of the declination of the poles of the orbs that carry the apogee, intersects the plane of the ecliptic in a diameter of the world. Consequently one of the parts of the plane of the eccentric will decline from the ecliptic toward the north, the other toward the south. Therefore that intersection of the circumference of the moon's eccentric with the plane of the ecliptic, the one that begins to move toward the north when the center of the epicycle is on it, is called the head of the dragon, the other one the tail [Figure 8]. These intersections, however, move daily beyond the diurnal motion about three minutes toward the west, by virtue of the motion of the orb that entirely surrounds the moon's other three orbs. Accordingly, the mean longitude of the head of the dragon of the moon is the arc of the zodiac reckoned westward from the beginning of Aries up to the line drawn from the center of the world through the section of the head.

But the true longitude of the head is the arc of the zodiac reckoned eastward from the beginning of Aries to the line already mentioned. The same can be said concerning the tail. From this it is clear that, by subtracting the mean longitude of the head from twelve signs, its true longitude is the remainder. And thus the common maxim, that the head of the moon goes as much with the mean motion against the firmament as in reality it goes with the firmament, is understood to mean that the mean longitude of the head of the moon extends westward to the point to which the true longitude extends eastward.

**ON THE THREE SUPERIOR PLANETS**

Each of the three superior planets has three orbs, divided from each other following the pattern of the three orbs of the sun [Figure 9]. In the middle orb, however, which is absolutely eccentric, each has an epicycle, to which (as in the case of the moon) the body of the planet is attached.

38 See, e.g., Ptolemy's table (Almagest 5.8, p. 238). For a given value of the true argument (found in cols. 1 and 2) the equation of argument (when the epicycle center is in the apogee of the eccentric) is given in col. 4.

39 For a given position of the centrum lunae (found in cols. 1 and 2), the proportional parts are given in col. 6 (Almagest 5.8, p. 238).

40 The variation of the diameter (diversitas diametri) is the increase in the equation of argument when the epicycle center moves from apogee to perigee of the eccentric for the same true argument. For a given value of the true argument (defining the moon's position in the epicycle), found in Ptolemy's table, cols. 1 and 2 (Almagest 5.8, p. 238), the variation in the equation of argument is given in col. 5. The product of the quantities found in cols. 6 and 5 is added to the equation of argument found in col. 4 to give the equation of true argument for the chosen position of the epicycle. The equation of true argument is then added to or subtracted from the mean longitude, found from the tables (Almagest 4.4, pp. 182–187), to give the true longitude of the moon. For a mathematical analysis of Ptolemy's derivation of the equation of true argument see Pedersen, Survey, pp. 192–199.

41 Cf. n. 29 above. The head and tail are the intersections of circles shown in the diagram; the circle through the center of the epicycle represents the eccentric and the other circle represents the ecliptic.
Moreover, the orbs carrying the apogees, by virtue of the motion of the eighth sphere, move on the axis and poles of the ecliptic [Figure 10]. But the orb carrying the epicycle moves eastward on its axis, which intersects the axis of the zodiac, and its poles are separated from the poles of the zodiac by unequal distances. For this reason it follows that the apogees of the eccentrics never pass through the ecliptic but always remain away from it toward the north, and the perigees toward the south—in such a way that the apogees of the orbs carrying the epicycles, likewise the perigees, and also the centers and poles of the eccentric deferents, describe circumferences parallel to the plane of the ecliptic by virtue of the motion of the eighth sphere. Consequently in those planets also, the planes of the eccentrics will be cut unequally by the plane of the ecliptic, and the greater parts will be left toward the apogee, the smaller toward the perigee.

The motion of the deferent of the epicycle about its center and its poles is nonuniform. Nevertheless this nonuniformity has this rule of regularity: that the center of the epicycle moves uniformly about a certain point in the line of the apogee, as far from the center of its orb as this center is distant from the center of the world. Consequently that point is called the center of the equant, and the circle imagined about that point, the same size and in the same plane as the deferent, is called the eccentric equant. Therefore it follows that the opposite of what happens in the case of the moon happens in the case of these three planets, namely, that the center of the epicycle moves slower to the extent that it is nearer the apogee of the deferent, and faster to the extent that it is nearer the perigee. The epicycle has two motions, of which one is in longitude and the other in latitude. Concerning the second something will be said later. But its motion in longitude is the one by which the body of the planet attached to it moves about its center, as it is carried eastward in the upper part and westward in its lower part. Hence in this matter the opposite happens to what happens with the epicycle of the moon. The axis of this motion lies transversely.

42 As shown in Fig. 10, the north poles of the deferent and ecliptic are closer together than the corresponding south poles.
43 Ita, ut auge, scilicet defferentium epicycos, similiter opposita, atque centra, et poli defferentium eccentricorum circumferentias, superficiell ciectpiae (virtute motus octavae sphaeae) descriptant aequidistantes.
44 In Fig. 10 these circles are represented by the straight lines drawn from the poles and apsides of the deferent parallel to the diameter representing the plane of the ecliptic.
45 Whereas in the case of the moon the motion of the epicycle is retrograde, in the cases of the superior planets it is direct.
versely over the circumference, sometimes parallel to the axis of the ecliptic and
sometimes not, as will appear. And it is irregular about the center of the epicycle.
But the irregularity has this rule: that the distance of the body of the planet from the
point of the mean apogee of the epicycle, whatever it may be, follows a regular
pattern. Likewise, therefore, in the case of these planets as in that of the moon, it
follows necessarily that the mean apogee of the epicycle and the true apogee as well
change continually; moreover, the motion of revolution of the epicycle on its center
is faster through the upper half of the deferent but slower through the lower half.
The revolution of the epicycle has this measure: it revolves once precisely in as
much time as it takes from the mean conjunction of the sun and the planet to the
next following, so that in every mean conjunction the center of the body of the [4.2v]
planet is in the mean apogee of the epicycle, and accordingly in every such oppo-
sition it will be in the perigee of the epicycle. It follows therefore that the center of
the body of the planet is always as many degrees and minutes distant from the mean
apogee of the epicycle as the line of mean longitude of the sun is distant from the
line of mean longitude of the planet. Therefore by subtracting the mean longitude of
the planet from the mean longitude of the sun, the mean argument of the planet must
remain. Hence it seems to happen, that as the center of the epicycle of the planet
goes around more slowly, its epicycle revolves more quickly by the same amount.
On account of such slowness, the mean conjunction of the sun with the planet
returns more quickly. Again the mean longitude of any of the three planets, added to
its motion in the epicycle, becomes equal to the mean longitude of the sun in
degrees and minutes.

The mean apogee of the epicycle is shown by the line drawn from the center of the
equant through the center of the epicycle [Figure 11]. But the true apogee is shown
by the line from the center of the world through the center of the epicycle. There is

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46 Hence the planet moves with constant angular velocity relative to the mean apogee of the epicy-

47 Thus the epicycle revolves in the mean synodic period.

48 Here the translation follows the Vienna Codex 5203. The printed text has coniunctio media
motus solis cum eo ("the mean conjunction of the longitude of the sun with [the planet]").

49 I.e., its argument, which Peurbach defines later.

50 This relation is established by Ptolemy in Almagest 10.6, pp. 480–484. Interpreted geometrically,

it means that the line joining the epicycle center to the planet is parallel to the line joining the earth to

the mean sun (Pedersen, Survey, pp. 283–285). The motion in the epicycle is a reflection of the annual

motion of the earth around the sun in the heliocentric theory.
no separation between these in distance along the epicycle when the center of the epicycle is in the apogee or perigee of the deferent. They differ in fact the most when near the mean distances of the deferent, which are determined by the line from the center of the eccentric deferent drawn perpendicular to the line of the apogee.\footnote{This line is not drawn in Fig. 11, but the epicycles are shown in the position of the mean distances and also in the apsides. In this diagram the outer circle represents the ecliptic.}

The apogee of the planet, in the second meaning, is the arc of the zodiac from the beginning of Aries up to the line of the apogee.

The line of mean longitude of the planet or epicycle is that drawn from the center of the world to the zodiac parallel to the line going from the center of the equant to the center of the epicycle.

The line of true longitude of the epicycle is the line that goes from the center of the world through the center of the epicycle to the zodiac.

The line of the true position or longitude of the planet is that drawn from the center of the world through the center of the body of the planet to the zodiac.

The mean longitude of the planet or the epicycle is the arc of the zodiac from the beginning of Aries eastward up to the line of mean longitude of the planet. However, the true longitude of the epicycle is the arc up to the line of true longitude of the epicycle. But the true longitude of the planet is calculated up to the line of true longitude of the planet.

The mean centrum of the planet\footnote{centrum medium planetae. Cf. centrum lunae.} is the arc of the zodiac from the line of the apogee to the line of mean longitude of the epicycle.

The true or corrected centrum is reckoned from the line of the apogee up to the line of true longitude of the epicycle.

The equation of center in the zodiac is the arc of the zodiac between the line of mean longitude of the epicycle and the line of its true longitude. This equation is zero when the epicycle is in the apogee or perigee of the deferent, greatest, however, when it is at the mean distances. Moreover, when the mean centrum is less than six signs, it is more than the true centrum; similarly the mean longitude of the planet is more than the true longitude of the epicycle. For this reason, the equation of center in the zodiac is then subtracted from the mean centrum, and also from the mean longitude of the epicycle, so that the true centrum and the true longitude of the epicycle remain. The opposite happens, however, when the mean centrum is more than six signs.

The equation of center in the epicycle is the arc of the epicycle, interposed between its mean apogee and true apogee. Like that in the zodiac, this is zero when the center of the epicycle is in the apogee or perigee of the deferent, greatest, however, at the mean distances of the deferent. Now, whatever the ratio is of the equation of center in the zodiac to the whole zodiac, that is the ratio of the equation of center in the epicycle to the whole epicycle, because the angle of one equals the angle of the other, on account of parallel lines. Therefore one alone is given in the tables, and the other is known. When, however, the equation of center in the zodiac is taken from the mean centrum, so that the true centrum is obtained, the equation of center in the epicycle is added to the mean argument for the purpose of obtaining the true argument. And conversely, when the former [the equation of center in the zodiac] is added, the other [the equation of center in the epicycle] is subtracted. For alternately they exceed each other and are exceeded by the same amount.

The mean argument of the planet is the arc of the epicycle reckoned from the mean apogee, following its motion, to the center of the body of the planet. But the true argument is calculated from the true apogee.

The equation of argument is the arc of the zodiac interposed between the lines of the true position of the planet and the true position of the epicycle. Just as in the case of the moon, this is zero when the center of the body of the planet is in the true apogee or perigee of the epicycle. It is greatest, however, when the body of the planet is in the line drawn from the center of the world at a tangent to the circumfer-
ence of the epicycle when the center of the epicycle is in the perigee of the deferent. When in fact the corrected argument is less than six signs, the line of true longitude of the planet precedes the line of true longitude of the epicycle; for this reason the equation of argument is then added to the true longitude of the epicycle to obtain the true longitude of the planet. And the opposite is the case when the corrected argument is more than six signs. It happens, however, that the equations of argument in the case of these planets, as in the case of the moon, are varied because of the approach of the center of the epicycle to the center of the world. Accordingly the equations of the separate arguments are greater when the center of the epicycle is in the perigee of the deferent than when it is in the mean distances of the deferent. In the latter case they are again greater than when the center of the epicycle is in the apogee of the deferent, always comparing equations to their corresponding arguments.

Therefore the excesses of the equations of arguments, when the center of the epicycle is in the mean distance of the deferent, over the equations when it is in the apogee are called the outer variations of the diameter, or those that pertain to the outer distance. But the excesses of these equations, when the center of the epicycle is in the perigee, over those in the mean distances are called the inner variations of the diameter, or those that pertain to the inner distance. This is in fact because the line extended from the center of the world to the apogee of the deferent is longer than the line taken from the same center to the mean distance of the deferent; however, the excess of the one over the other is divided into sixty equal parts, called the outer proportional parts or those that pertain to the outer distance. Therefore the line of the true longitude of the epicycle, when it is in the apogee of the deferent, has all of these parts inside the circumference of the deferent; but in the mean distances, all outside but none inside. In the intermediate positions, however, the line of the true longitude has some inside and some outside, and as many more inside as the center of the epicycle is nearer to the apogee of the deferent. Similarly, the line extended from the center of the world to the mean distance of the deferent is longer than the line taken from the same center to the perigee of the deferent. The excess, however, of the former over the latter is divided into sixty parts, called the inner proportional parts, or those that pertain to the inner distance. Therefore the line of true longitude of the epicycle, when it is in the mean distance, has none of them outside the circumference of the deferent, but all of them when it is in the perigee. In intermediate places, however, it has as many more outside as the center of the epicycle is closer to the perigee.

The equations of the arguments, which are written in the tables, happen when the center of the epicycle is disposed in the mean distances of the deferent. But the latter (as has been said) are greater than those that occur when it is in the apogee, and less than the others that occur in the perigee. When therefore the center of the epicycle is outside the mean distance of the deferent, the proportional parts are known by means of the true centrum, and the variation of the diameter is found by means of the true argument—the variation is in fact greater if the proportional parts are the outer parts, less, however, if they are the inner parts. The proportional part of this variation, following the ratio of proportional parts to sixty, is added to the equation of argument found in the tables, or subtracted from it. It is in fact added if
the variation is inner, subtracted if outer, and the equation of argument, both true and corrected, is produced, corresponding to the appropriate position of the center of the epicycle.

ON VENUS

Venus has three orbs with an epicycle, disposed as to position and motion in longitude like those of the superior planets. For the orbs carrying the apogee move on the axis of the zodiac, following the motion of the eighth sphere—in such a way, however, that the apogee of its eccentric is always under that place of the zodiac under which the apogee of the eccentric of the sun lies. Accordingly, when the apogee of the sun in the second meaning is known, the apogee of Venus is known at the same time.

Now the orb carrying the epicycle has two motions: one proceeds in longitude toward the east, uniformly about the center of the equant, as in the superior planets—in such a way, however, that the center of the epicycle makes one revolution precisely in the time in which the orb carrying the sun makes one. For Venus behaves toward the sun in such a way that the line of its mean longitude terminates in the same point of the zodiac in longitude as the line of mean longitude of the sun. Hence, as we have the mean longitude of the sun, the mean longitude of Venus is obtained. Therefore they are always in their mean conjunction.

The motion of the deferent in longitude is made about its imaginary axis, whose poles approach and recede from the poles of the zodiac on each side, on account of the other motion of the eccentric in latitude, concerning which something will be said later. Therefore Venus does not resemble the superior planets, in which the apogee of the eccentric does not cross the ecliptic, but, as will appear, in truth it sometimes declines to the south and sometimes to the north. But its epicycle moves with a double motion, namely in longitude and in latitude—in longitude, in fact, just like the epicycles of the superior planets. Nevertheless it always revolves almost once in nineteen solar months.\(^59\) Hence in this motion it does not look to the sun, as the superior planets do.\(^60\) The definitions of terms are here throughout just as for the three superior planets.\(^61\)

ON MERCURY

Mercury has five orbs and an epicycle [Figure 13]. The two outermost orbs are eccentric relatively. For the convex surface of the highest and the concave surface of the lowest are concentric to the world, but the concave surface of the highest and the convex surface of the lowest are eccentric to the world; nevertheless they are concentric to each other. And their center is as distant from the center of the equant as the center of the equant is from the center of the world. And it is the center of the small circle,\(^62\) which the center of the deferent describes, as will be seen. Now they are called the deferents of the apogee of the equant, and they move with the motion of the eighth sphere on the axis of the zodiac.

\(^{59}\) This is the synodic period of the planet; see Almagest 9.3, p. 425.
\(^{60}\) In the case of Venus it is the motion of the deferent (and not that of the epicycle) that reflects the motion of the earth in the heliocentric theory.
\(^{61}\) The method of deriving the true longitude from Ptolemy's table (Almagest 9.4, 9.11, pp. 436–438, 552) is the same as that for the superior planets.
\(^{62}\) This circulus parvus, similar to that in the lunar theory, was introduced by Ptolemy to vary the distance of the epicycle center from the earth in such a way that the double perigee of Mercury could be explained.
Between these outer orbs are two others similarly of nonuniform thickness, and inside these is located the fifth orb, namely, the one that carries the epicycle. For the convex surface of the higher and the concave surface of the lower have the same center as the small circle, but the concave of the higher and the convex of the lower, together with both surfaces of the fifth orb, have another moving center, which is called the center of the deferent. These two orbs are called the deferents of the apogee of the eccentric. And they move uniformly on the center of the small circle westward, with such speed that, exactly in the time that the line of mean longitude of the sun makes one revolution, these orbs likewise complete one in the opposite direction. This motion is made on an axis sometimes parallel to the axis of the zodiac and passing through the center of the small circle [Figure 14]. However, it follows from the motion of these orbs that the center of the orb of the deferent of the epicycle likewise describes in the same time a certain circumference of a small circle. The radius of this circle in fact is as great as the distance separating the center of the equant from the center of the world. Accordingly, this circumference will pass through the center of the equant.

But the fifth orb carrying the epicycle, located within the two subordinate orbs, moves in longitude eastward, carrying the center of the epicycle uniformly about the center of the equant, which is indeed in the middle between the center of the world and the center of the small circle. But the fifth orb has such speed that the center of the epicycle revolves once in the same time that the line of mean longitude of the sun completes one revolution. For Mercury in this respect behaves like Venus toward the sun. For it always happens that the mean longitude of the sun is also the mean longitude of these two planets.

Therefore from these things and what has been said above, it is evident that the six planets share something with the sun in their motions and that the motion of the sun is like some common mirror and rule of measurement to their motions.

Now the motion of the orb of this planet which carries the epicycle is made about

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63 Following Ptolemy's explanation of the motion in latitude, Peurbach supposes that the plane of the deferent of the epicycle has a variable inclination (denoted by the technical term "deviation") to the plane of the ecliptic, so that the plane of the deferent rocks about the nodes and the inclination is zero when the center of the epicycle is in one of the nodes. The diagram illustrates an instant when the inclination is zero. In this case the axis of the deferent (and also that of the deferents of the apogee) is parallel to the axis of the ecliptic. As the center of the deferent describes the small circle, its axis remains parallel to the axis of the deferents of the apogee and rotates about it.
an imaginary axis whose extremities, as appeared in Venus, on account of the other
motion that it has in latitude, similarly approach the poles of the zodiac and recede
from them.\footnote{The mean eastward motion of the deferent of the epicycle, in a period of one tropical year, is
equal and opposite to the mean westward motion of the deferents of the apogee. The two motions
take place about parallel axes.} However, this axis as a whole is mobile following the motion of the
center of the deferent in the small circle.

It appears therefore that just as in the moon the center of the epicycle passes
through the deferents of the apogee of the eccentric twice in a lunar month; so in
Mercury the center of the epicycle passes through the deferents of the apogee of the
deerent of the epicycle twice in a year. But it is only once in the apogee of the
deerent.

For the apogee of the deferent of Mercury does not move circularly, completing
circular revolutions as happens in the case of the moon. Rather, on account of the
motion of the center of the deferent in the small circle, it proceeds at one time
eastward and at another time westward. For there are certain limits that the apogee
of the deferent does not have the power to overstep in receding from the apogee of
the equant; but it continuously rolls forward and rolls back, ascending and descend-
ing under the arc of the zodiac formed by two lines drawn tangent to the small circle

from the center of the world to the zodiac [Figure 15\footnote{In Fig. 15 the two large circles show the deferent in its apogee and perigee positions. The apogee
and perigee move in the lune-shaped figures shown between the circles. Cf. Fig. 23 in Willy Hartner, Orients-occi
dens (Hildesheim: Olms, 1968), p. 489, which interprets Peurbach’s Fig. 15, showing the
curves traced out by the apogee and perigee of the deferent. Although Peurbach seems to have been
the first to give attention to these curves, it is odd, as Hartner remarks, that he does not show them
as smooth curves.}]. As often as the center of the
epicycle is in the apogee of the deferent, it will also be, by the similitude of motions,
in the apogee of the equant, and the center of the deferent will be in the apogee of its
small circle.\footnote{I.e., the point on the small circle diametrically opposite the center of the equant.} For this reason, the center of the epicycle will then be at its greatest
distance from the center of the world, and the center of the deferent will be twice as
far from the center of the equant as the center of the equant from the center of the
world. Thereafter, when the center of the deferent moves by the motion of the two
secondary orbs from the apogee of its circle toward the west, the center of the
epicycle will move by the motion of the deferent from the apogee of the equant just
the same amount toward the east.\footnote{See Pedersen, Survey, p. 316, Fig. 10.7.} Accordingly, the center of the deferent begins to
move toward the center of the world, and the apogee of the deferent recedes contin-
ually from the apogee of the equant toward the west, until the center of the deferent

is in the western tangent to the [small] circle.\footnote{The positions of the epicycle in this case and in that in which the center of the deferent is in the
eastern tangent of the small circles are shown in Fig. 15. Their centers are determined by marking off
the radius of the deferent from the points of contact of the tangents to the small circle along the lines
making angles of 120° with the line of the apogee.} This happens, moreover, when the
center of the deferent is four signs distant from the apogee of the small circle. And
then similarly the center of the epicycle will be four signs distant from the apogee of
the equivant toward the east. Furthermore, the apogee of the deferent will be in its

\[4.7r\]

greatest separation from the apogee of the equant toward the west. And in this
position the center of the epicycle will be in the closest approach it is accustomed
to make to the center of the world. However, it will not then be in the perigee of the
deerent, nor in the line tangent to the small circle drawn through the center of the
world. For afterward the center of the deferent descends toward the center of the
equart, and the apogee of the deferent begins to move again toward the apogee of
the equant; but the center of the epicycle descends proportionately in the other half
toward the perigee of the equant. Hence it moves farther from the center of the
world, and it does not arrive at the perigee of the deferent except when this is in the
perigee of the equant. This happens, however, when the center of the deferent
arrives at the center of the equant. And then the apogee of the deferent will also be
with the apogee of the equant; both deferent and equant, inasmuch as they are
constituted equal in size, will be a single circle; and the center of the epicycle will

\[4.6v\]
then be more distant from the center of the world than it was when in a position four signs from the apogee of the equant. Then again when the center of the deferent recedes from the center of the equant by ascending in its circle, the center of the epicycle will recede from the perigee of the equant and of the deferent and will continually approach closer to the center of the world. But the apogee of the deferent will move away from the apogee of the equant toward the east continually until the center of the deferent arrives at the line tangent to the small circle on the eastern side. This point of contact is also four signs distant toward the east from the apogee of the small circle. For then the apogee of the deferent will have its maximum separation toward the east from the apogee of the equant. And the center of the epicycle will again be in its closest approach to the earth that it is accustomed to have; it will not, however, be in the perigee of the deferent. From this place indeed, as the center of the deferent ascends toward the apogee of the small circle, the apogee of the deferent will continually return to the apogee of the equant. And the center of the epicycle will separate more from the center of the world toward the apogee of the equant, ascending up to the moment when the center of the deferent arrives at the apogee of the small circle. For then the apogee of the deferent will coincide with the apogee of the equant, and the center of the epicycle will be similarly in the apogee of the deferent just as it is in the apogee of the equant. Consequently it will be a second time in the maximum separation from the center of the world, just as at first. And again thereafter it follows the same sequence of changes, as has just been explained.

From these things is seen, first, that in a year the center of the deferent is only once the same as the center of the equant. At all other times, however, the center of the deferent is farther from the center of the world than the center of the equant is. For this reason, the opposite follows to what happens in the case of the superior planets and Venus, namely, the nearer the center of the epicycle is to the apogee of the equant, so much the faster it moves, and the nearer to the perigee, so much the slower.

Second, although the center of the epicycle will be at the greatest distance from the center of the world only once in a year, it still happens to be twice at its closest [Figure 16]. Similarly, although it is at its closest approach twice in a year it is found in the perigee of the deferent only once in a year.

69 The curve described by the center of the epicycle is shown in Fig. 16, though in the apogee and the point diametrically opposite it should be shown as smooth.
Third, when the center of the epicycle is outside the apogee or perigee of the equant, the perigee of the deferent must always be located between the center of the epicycle and the perigee of the equant, turning itself sometimes toward the center of the epicycle and sometimes away from it, both when it is preceding and when it is following [the center of the epicycle].

Fourth, just as the apogee of the deferent moves away from the apogee of the equant to certain limits on both sides, the perigee of the deferent has the same relation to the perigee of the equant. The arc of this kind of motion of the apogee of the deferent is greater, however, than the arc of the motion of the perigee. Therefore the motion of the one will be faster than the motion of the other.

Fifth, even though the center of the epicycle may [at some time] happen to be in the point of the deferent most remote from the center of the world, it is at no time in the point of the deferent that happens to be closest to the center of the world. For when the center of the epicycle is in the apogee of the deferent, the situation of the deferent is such that its perigee is then so near the center of the world that in every other situation of the deferent that arises, no point of it is found nearer or as near to the center of the world. The center of the epicycle is not in the point, however, that happens to be nearest, when that point in fact happens to be closest, but in its opposite.70

Sixth, from what has been said it appears clearly that the center of the epicycle of Mercury, on account of the motions stated above, does not, as in the cases of the other planets, describe the circular circumference of the deferent but rather the periphery of a figure that resembles a plane oval.71 The epicycle of Mercury in fact moves in longitude just as the epicycle of Venus does. Nevertheless it completes one revolution on its center in nearly four solar months. However, the terms of the tables are shown here just as for the superior planets; except that differences exist in some proportional parts. For the equations of arguments of Mercury that are written in the tables are those that occur when the center of the epicycle is in the middle of its separation from the earth. This happens when the center of the epicycle is distant from the apogee of the equant by two signs, four degrees, and thirty minutes. But in the case of the other planets this happens when the center of the epicycle is in the mean distances of the deferent. In the same way, the minimum distance of the center of the epicycle of Mercury from the center of the world occurs when the center of the epicycle is four signs distant from the apogee of its equant. And this happens in the other planets when the center of the epicycle is in the perigee of the equant.72 Therefore the outer proportional parts are the excess of the maximum distance of the center of the epicycle over its mean distance divided into sixty equal parts. But the inner proportional parts are defined as the excess of the mean distance of the center of the epicycle over its minimum distance similarly divided into sixty equal parts. And following this the twofold variation of the diameter is defined. Because, however, from the position of closest approach of the center of the epicycle toward the perigee of the equant the inner proportional parts diminish—though previously from the position of mean distance up to the position of closest approach they continually increased—Mercury is said to have proportional parts in three modes;73 but in the cases of Venus and the three superior planets they are in two

70 I.e., the epicycle center is in the apogee of the deferent when the perigee of the deferent is at its closest point to the center of the world. When the epicycle center is in the point of the deferent opposite the apogee, however, this point is farther from the center of the world.

71 A modern analysis of the motion of the epicycle in the theory of Ptolemy described by Peurbach may be found in Hartner, Oriens-occidens, (cit. n. 65), pp. 465-478. Peurbach was the first European to describe the curve as similar to an ellipse, though it had been so described by al-Zarqālī in the eleventh century. According to Hartner’s analysis, the curve implied by Ptolemy’s theory is practically an ellipse. On al-Zarqālī see Heinrich Suter, Die Mathematiker und Astronomen der Araber und ihre Werke (Leipzig: Teubner, 1900; rpt. New York: Johnson, 1972), pp. 109–111.

72 Thus the path of the center of the epicycle of Mercury has two perigees separated from its apogee by 120°.

73 For a given value of the true argument (i.e., the position of the planet in its epicycle), the equation of argument, assuming the epicycle center to be in its mean distance, is given in col. 6 of
ON THE DIVERSE PHENOMENA OF THE PLANETS

A planet is said to be direct when the line of its true longitude advances eastward, but retrograde when it advances westward, and stationary when this line is seen to stand still. The first station in the first meaning is the point of the epicycle at which the planet will be when it begins to be retrograde. The second station in the first meaning is the point of the epicycle at which the planet will be when it begins to be direct. In fact, when the center of the epicycle is in the same position of the deferent, these stations occur on both sides equidistant from the true perigee of the epicycle [Figure 17].

The first station in the second meaning is the arc of the epicycle between the true apogee of the epicycle and the point of the first station.

The second station in the second meaning is the arc of the epicycle from the true apogee, through the perigee, up to the point of the second station.

The arc of the direct motion is the arc of the epicycle from the second station, through the apogee, up to the first station in the first meaning.

But the arc of retrogradation is the arc of the epicycle from the point of the first station, through the perigee, up to the point of the second station. These arcs are in fact increased and decreased on account of the variation of the aforementioned points. For the center of the epicycle is nearer the perigee of the equant to the same extent as the points of the stations are nearer the true perigee of the epicycle. This same relation holds true in proportion as the planet has a larger epicycle and slower motion of argument. And hence the periods of direct motion and retrogradation also vary in their quantities. For such a period results when its arc is divided by the motion of argument of the planet in one day.

From what has been said it follows that, if the first station is subtracted from the whole circle, the second station remains, but by subtracting the first station from the

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Ptolemy's table (Almagest 9.11, p. 553). Cols. 5 and 7 give the differences (for a given true argument) when the epicycle center is in the apogee or perigee of its oval path. Col. 8 gives the proportional parts to be used in the interpolations for other positions. But in the case of Mercury, different interpolations are needed over three sections: from apogee to mean distance, from mean distance to perigee of the oval (120° from the apogee), and from perigee of the oval to the position opposite the apogee (i.e., the perigee of the equant). For Ptolemy's explanation of the construction of the tables see Almagest 11.10, pp. 545-548. Cf. Pedersen, Survey, pp. 326-328.
second, the arc of retrogradation will be obtained, which, if taken from the whole
circle, leaves the arc of the direct motion. However, the moon, unlike the other
five planets, neither has a station nor is retrograde—although it has an epicycle—
because of the speed of the center of its epicycle. For the center of the moon’s
epicycle in any day always describes a greater arc of the zodiac eastward than is the
arc of the zodiac corresponding to the arc of the epicycle that the center of the body
of the moon traverses in any given day westward in the upper part of the epicycle.
Nevertheless, when in the upper half of the epicycle the moon is slow, and in the
[5. lv] lower it must progress swiftly.

The planets are said to be slow and reduced in pace when their line of true
longitude is slower than the line of mean longitude, or proceeds westward. On the
other hand, they are said to be fast and increased in pace when they move faster
than the mean motion eastward. They are said to be increased in number when the
equation of argument is added to the mean longitude, reduced when it is subtracted;
increased in light when they recede from the sun or the sun from them, reduced in
light when they approach to the sun or the sun to them. They are said to be eastern
and matutinal when they rise before the sun, to be western and vespertine when
they set after the sun.

Those planets become visible by a morning rising which begin to appear early in
the morning before the rising of the sun, emerging from under the rays on account of
their retreating from the sun or the sun’s from them. Those become visible by an
evening rising which begin to appear after the setting of the sun, coming out from
under the rays on account of their retreating from the evening sun.

Those planets go down with a morning setting which enter the sun’s rays and on
account of their approach to the sun begin to disappear early in the morning. Those
go down with an evening setting which enter the sun’s rays and, on account of their
approach to the sun or of the sun’s to them in the evening after the sun has set,
begin to disappear. The three superior planets neither go down with a morning
setting nor come out with an evening rising, but Venus, Mercury, and the moon do.

There are three reasons why the moon after its conjunction with the sun some-
times appears sooner and sometimes later. One reason is the inclination or obliquity
of the zodiac and horizon. For if there is a conjunction under the ecliptic—but it is in
the half between the end of Sagittarius and the end of Gemini—then when the sun is
setting on the horizon, there will be more degrees in the circle of revolution of the
moon from the moon to the horizon than from the moon to the sun with respect to
the zodiac. Accordingly, in the northern climes it will be visible sooner than if it had
been in the other half of the zodiac. The second reason is the latitude of the moon
from the ecliptic. For if after the conjunction the moon moves in a northern latitude,
it will again be visible sooner than if it were moving in a southern latitude. The third

74 Peurbach gives here only a short descriptive account of the phenomena of stations and retrogra-
dations; he does not even mention the greatest elongations of Venus and Mercury from the sun.
Ptolemy devotes a whole book of the Almagest to these phenomena; Almagest 12, pp. 555–596; cf.
Pedersen, Survey, pp. 329–354. The reader of Peurbach, however, should be in a position to use the
tables of stations, since the stations are given as functions of the mean centrum, while the table of
elongations is self-explanatory.

75 minuti cursu. This is the first of a number of medieval technical terms relating to the phenomena
of the planets introduced by Peurbach.

76 aucti curso.

77 The planet is said to be aucti numero when the equation is added and minuti numero when the
equation is subtracted. These terms are used in the Theorica planetarum; see Theorica planetarum

78 The planet is said to be aucti lumine when it recedes from the sun and minuti lumine when it
approaches the sun. These terms also are used in the Theorica planetarum.

79 orientales et matutini. These terms are of Greek origin. See, e.g., Ptolemy, Tetrabiblos 1.6 (Loeb

80 occidentales et vespertini. Again these terms are of Greek origin. They may be found in Ptolemy,
Tetrabiblos 1.6.

81 See Ptolemy’s tables of first and last visibilities of the planets; Almagest 13.10, p. 647.
The reason is the speed of the true motion of the moon. For if the moon is fast in motion, it appears sooner than if it is slow. Therefore it happens sometimes that all these causes concur; then the moon appears old and new on the same day. But sometimes no more than two of the causes concur; then it is seen on the second day after conjunction. Sometimes indeed only one is present; then it is seen on the third day. Again, sometimes the opposite of all of them is the case; then it happens that the moon appears on the fourth day.

The aspect of the planets is trine when their true places are separated by the third part of the ecliptic, quartile when by the fourth part, sextile when by the sixth part. A mean conjunction of the planets takes place when their lines of mean longitude are united according to longitude of the zodiac. But a true conjunction takes place when the lines of true longitude thus come together. A visible conjunction, however, takes place when the lines taken from our eye through the centers of the bodies are united into one. Similar things can be said concerning the mean and true oppositions. And they are designated in the same signs—degrees and minutes. From this it appears that the true conjunction often occurs when the mean has preceded it or is yet to be; often again the true conjunction occurs when the visible nevertheless does not. Sometimes again the visible conjunction precedes the true, but at other times it follows.

The true position of a star is the point of the firmament terminating the line extended from the center of the world through the center of the star. But the

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\[82\] Ptolemy's reference in the *Almagest* to these astrological configurations is equally brief (*Almagest* 8.4, p. 407). However, he wrote an influential treatise on astrology, the *Tetrabiblos*, in which he endeavored to give this science a rational basis. The diagram illustrating the aspects is missing from Peurbach's manuscript. See, e.g., Vienna Codex 5203, fol. 16v.

\[83\] *Coniunctio media planetarum fit, quando lineae mediorum motuum eorum secundum longitudinem zodiaci coniunguntur*. Mean conjunctions are shown at the top and bottom of Fig. 19, where the dot on the deferent of the outer planet indicates the center of its epicycle. True conjunctions are shown at the sides, where in each case the dot represents the body of the planet.

\[84\] According to Ptolemy, neither the fixed stars nor the five ordinary planets show any appreciable parallax. In the case of the moon, however, consideration of the parallax is an essential element in the theory of eclipses.

\[85\] In the first diagram, the smallest semicircle represents the surface of the earth. The observer is situated at the top of this semicircle.
visible or apparent position is determined by the line taken from the eye through the center of the star.

The parallax of a star is the arc of the great circle passing through the zenith and the true position of the star intercepted between the true and apparent positions of the star. Consequently it is clear that, to the extent that the star will be nearer to the center of the world and the horizon, so much greater will be the parallax. It is also clear that the parallax is found to be greatest in the moon, but in Mars is not easily perceived. For the radius of the earth is sensible in magnitude compared to the radius of the orb of the moon but is not very sensible compared to the radius of the orb of Mars.

The parallax of a star in longitude is the arc of the ecliptic intercepted between two great circles, of which one passes through the poles of the ecliptic and the true position but the other through the poles and the apparent position of the star.

The parallax of a star in latitude is the arc of the great circle passing through the poles of the zodiac and the true position of the star that is intercepted between two circles parallel to the ecliptic, of which one passes through the true position of the star and the other through its apparent position. However, the arc of each of these circles parallel to the ecliptic that is intercepted between the great circles passing through the poles of the zodiac is similar to the parallax in longitude. Consequently the parallax is as it were a diagonal line of the rectangle whose sides are the parallaxes in longitude and latitude.

The parallax of the moon in relation to the sun is the excess of the parallax of the moon over the parallax of the sun. If the true conjunction of the luminaries is between the ascendant degree of the ecliptic and the ninetieth degree of that from the ascendant, their visible conjunction has preceded the true. If however it is between the same ninetieth degree and the descendant degree, the visible conjunction will follow the true. But if this happens in that same ninetieth degree, then the visible conjunction will occur at the same time as the true, and no parallax in longitude will occur. For the ninetieth degree of the ecliptic from the ascendant is always in the circle passing through the zenith and the poles of the zodiac.

The apparent latitude of the moon is the arc of the great circle that passes through the poles of the zodiac and through the true or apparent position of the moon and is intercepted between the ecliptic and the circle parallel to it that passes through the apparent position.

The minutes of immersion in the case of an eclipse of the moon are the minutes of the zodiac that the moon traverses in exceeding the sun from the beginning of the eclipse to the middle of it, if it is partial or total without delay—or from the beginning up to total obscuration if it is total with delay.

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86 diversitas aspectus. The parallax theory is illustrated in the second diagram of Fig. 20.

87 This is the parallax in altitude; *Almagest* 5.17, p. 258; cf. Pedersen, *Survey*, pp. 213–214. Note that Peurbach is not concerned with refraction effects.

88 I.e., the total parallax (or parallax in altitude) is composed of the parallaxes in longitude and latitude added vectorially; see *Almagest* 5.18, pp. 264–273; and Pedersen, *Survey*, pp. 217–219.

89 diversitas aspectus lunae ad solem. Ptolemy’s *Handy Tables* include tables of the components in longitude and latitude of this difference, called “adjusted parallax” by Islamic astronomers; see O. Neugebauer, A *History of Ancient Mathematical Astronomy* (Berlin: Springer, 1975), p. 990. The adjusted parallax is of interest only in the case of solar eclipses. Cf. the parallax tables in *Almagest* 5.18, p. 265.

90 The ascendant (or horoscope) is the point of the ecliptic rising above the horizon. The descendant is the point of the ecliptic setting below the horizon; see Neugebauer, *History* (cit. n. 89), pp. 114–115. The degree of the ascendant is its longitude.

91 When the stars are on the great circle passing through the poles of the zodiac and the zenith (see Fig. 20), the parallax in longitude is clearly zero.


The minutes of half the delay\textsuperscript{94} are the minutes of the zodiac that the moon traverses in exceeding the sun [in speed] from the beginning of total obscuration up to the middle of the eclipse.

The minutes of immersion in the case of a solar eclipse [Figure 22] are the minutes that the moon completes from the beginning of the eclipse up to the middle, by its excess [of speed] over the sun. For this reason, if these minutes are divided by the [5.4v] excess [of motion] of the moon in an hour,\textsuperscript{95} the time in which it passes over them will be the result.

The apparent diameter of the sun in the apogee of the eccentric subtends thirty-one minutes, but in the perigee thirty-four minutes.\textsuperscript{96} Always, however, the [true] motion of the sun in an hour to its apparent diameter has the proportion of five to sixty-six.\textsuperscript{97} The moon, on the other hand, in the apogee of the eccentric and epicycle has an apparent diameter of twenty-nine minutes, but in the apogee of the eccentric and perigee of the epicycle an apparent diameter of thirty-six minutes. Always, however, the [true] motion of the moon in an hour to its apparent diameter has the proportion of forty-eight to forty-seven.\textsuperscript{98} For this reason it follows that it is possible for a total eclipse of the sun to occur at some time. Never, however, can it appear naturally, by reason of the parallax, that the whole sun is universally eclipsed in the whole of the earth. When the sun is in the apogee of the eccentric, the diameter of the shadow in the place of the transit of the moon is to the apparent diameter of the moon just as thirteen is to five.\textsuperscript{99} But the excess of the diameter of the shadow, 

\textsuperscript{94} \textit{minuta morae dimidiae}.

\textsuperscript{95} In fact the result of the division is half the duration of the eclipse.

\textsuperscript{96} Ptolemy supposed the diameter of the sun to be constant and of the magnitude 31' (\textit{Almagest} 5.14, p. 252).

\textsuperscript{97} This rule for finding the solar diameters stated by Peurbach goes back through al-BATTÀNÌ and al-Khwàrizmi to Indian astronomy; see N. M. Swerdlow and O. Neugebauer, \textit{Mathematical Astronomy in Copernicus's De revolutionibus} (Berlin: Springer, 1984), Pt. 1, p. 250. Cf. N. M. Swerdlow, "Al-Battànî's Determination of the Solar Distance," \textit{Centaurus}, 1973, 17:97–105. Al-Battànî gives the hourly motion of the sun in perigee as 2' 33" and in apogee as 2' 23". Taking the diameter in apogee as 31' 20", he used the rule to calculate the diameter in perigee and obtained the value 33' 40": \textit{Al-Battànî opus astronomicum}, ed. Carlo A. Nallino (Milan, 1899–1903; Frankfurt am Main: Minerva, 1969), Pt. 1, p. 58.

\textsuperscript{98} Again this rule is taken from al-Battànî: \textit{Al-Battâni}, ed. Nallino, p. 97. Ptolemy supposed the diameter of the moon to vary between 31' and 35' (\textit{Almagest} 5.14; 6.8, pp. 254, 284). This would preclude the possibility of an annular eclipse.

\textsuperscript{99} This relationship is found in Ptolemy, \textit{Almagest} 5.14, p. 254.
when the sun is in the apogee, over its diameter when the sun is elsewhere in the eccentric is ten times the difference of the motions of the sun by which it moves in an hour when it is in the apogee and in that other position.

**ON DECLINATION AND LATITUDE**

The declination\(^{100}\) of a star [Figure 23] is its distance from the equator and is reckoned in the circle passing through the poles of the world and the true position of the star, which the line drawn from the center of the world through the center of the body of the star defines.

But the latitude of a star is its distance from the ecliptic and is reckoned in the circle that passes through the poles of the ecliptic and the true position of the star as just described.

From these definitions and from what has been said above concerning the sun, it is clear that, although it has a declination, the sun has no latitude, because the plane of its deferent always remains in the plane of the ecliptic. But the moon and the other five planets have latitude. For in the case of the moon, on account of the inclination\(^{101}\) of the axis of the movement of the apogee to the axis of the zodiac, the plane of its deferent always cuts the plane of the ecliptic along a diameter of the world, inclining from the plane of the ecliptic on opposite sides by the amount of its maximum inclination, which always invariably remains the same. For the plane of the moon's epicycle never recedes from the plane of the deferent. Therefore the moon has only one latitude, namely, the one that arises on account of the inclination of the deferent to the ecliptic. And this is known through the true argument of latitude of the moon.

Accordingly, the mean argument of latitude of the moon is the arc of the zodiac taken eastward between the line of true longitude of the head of the dragon and the line of mean longitude of the moon.

But the true argument of latitude of the moon is the arc of the zodiac reckoned eastward from the line of true longitude of the head of the dragon to the line of true longitude of the moon. Therefore by subtracting the true longitude of the head from the true position of the moon, or by adding the true longitude of the moon to the mean longitude of the head, the true argument of latitude of the moon will be found.\(^{102}\)

The three superior planets, on the other hand, have a twofold latitude.\(^{103}\) One occurs because of the inclination of the plane of the deferent to the plane of the ecliptic on opposite sides; the maximum of this inclination, as in the case of the moon, always remains the same. However, the intersections of the deferents with the ecliptic on a diameter of the world, which also are called head and tail,\(^{104}\) do not, as in the case of the moon, move westward, but as has been said, follow the motion of the eighth sphere. It is thus that the apogees of those deferents always describe circumferences parallel to the ecliptic on the north side. Although, again, the apogees of those deferents are always north, in all three planets they are still not the points of the deferents' greatest latitude from the ecliptic. Indeed, only in the case of Mars does the apogee of the deferent decline in the highest degree to the north from the ecliptic. But in the case of Saturn such a point stands before the apogee of its deferent, namely, westward by fifty degrees. In the case of Jupiter, on the other hand, it stands after the apogee, namely, eastward by twenty degrees.\(^{105}\)

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\(^{100}\) *declinatio*. Ptolemy has no equivalent for this term in the *Almagest*.

\(^{101}\) In this context, Peurbach's *declinatio* has been translated as "inclination."

\(^{102}\) Ptolemy treats the lunar latitude in *Almagest* 5.9, pp. 237–239. A description of Ptolemy's calculation of the lunar latitude is given in Pedersen, *Survey*, pp. 200–202; and an example by Toomer may be found in *Almagest*, p. 652.\(^{102}\)

\(^{103}\) This arises from the inclination of the plane of the epicycle to the plane of the deferent. In the theory of Ptolemy, the plane of the epicycle is parallel to that of the ecliptic only at the nodes. This means that the Ptolemaic and Copernican systems of latitude cannot be geometrically equivalent; see Pedersen, *Survey*, pp. 364–365.

\(^{104}\) Peurbach also calls them nodes.

\(^{105}\) See *Almagest* 13.1, 6, pp. 597–598, 635. Cf. Pedersen, *Survey*, pp. 356–359. The point of maximum latitude is at the top of the eccentric, 90° distant from the nodes. In the case of Mars the top of
The other latitude, however, arises from the plane of the epicycle sometimes inclining to the plane of the deferent. For the epicycle moves in latitude with respect to the true apogee on its axis, which passes through its center and the mean distances—in such a manner, however, that when the center of the epicycle is in the node of the head or tail, the true apogee and perigee of the epicycle are directly in the plane of the deferent, and the plane of the epicycle is in the plane of the ecliptic. But after the center of the epicycle recedes from the node, the diameter of the apogee and perigee begins to incline to the plane of the deferent, so that the true perigee of the epicycle begins to move away from the plane of the deferent in the same direction as the half of the deferent through which the center of the epicycle then begins to move inclines to the ecliptic, and the true apogee of the epicycle moves away the same amount on the opposite side. And thus the apogee and perigee of the epicycle continually move away from the plane of the deferent, until the center of the epicycle arrives at the point of the deferent in the greatest inclination to the ecliptic, namely, halfway between the two nodes. There the plane of the epicycle, together with the diameter of the apogee and perigee, inclines in the greatest degree to the deferent. But from this place the inclination of the epicycle to the deferent is successively reduced until the center of the epicycle arrives at the other node, in which again the whole plane of the epicycle will be in the plane of the ecliptic, and the diameter of the true apogee and perigee in the plane of the deferent. Consequently, the axis on which this motion in latitude takes place, when the center of the epicycle is outside the nodes, is always parallel to the plane of the ecliptic.

From these statements it appears, first, that the axis (as said above) on which the eccentric coincides approximately with the apogee, but in the cases of Jupiter and Saturn the longitudes of the top and apogee of the eccentric differ by the quantities stated.

Besides the slow variation in latitude during one complete revolution through the zodiac, the superior planets also have a more rapid change in latitude in the periods around the retrogradation. To represent this change Ptolemy introduced a variable inclination of the plane of the epicycle to that of the deferent; see Pedersen, Survey, pp. 359–368.

When the epicycle is in one of the nodes, the diameter containing its true perigee and true apogee coincides with the line of nodes, while the perpendicular diameter is in the plane of the ecliptic. In the subsequent motion, remains parallel to the plane of the ecliptic and serves as an axis about which oscillates. As the epicycle center moves to northern latitudes, tilts so that the perigee moves toward the north. The variable angle of inclination of to the plane of the deferent is called the deviation.

In the case of all three superior planets the maximum deviation exceeds the inclination of the plane of the deferent (Almagest 13.3, pp. 604–605).

I.e., the diameter containing the true apogee and true perigee of the epicycle.
revolution of the epicycle in longitude takes place is sometimes parallel to the axis of
the ecliptic and sometimes not, but will never be parallel to the axis of the eccen-
tric.

Second, the body of the planet, when it is in the upper half of the epicycle with the
center of the epicycle outside the nodes, will always be between two planes,
namely, those of the ecliptic and of its deferent. But when it is in the lower half of
the epicycle, it will be farther from the ecliptic than the deferent is. Therefore the
star will not always be found between the deferent and the ecliptic.

Third, the true and mean apogees of the epicycles are not always the ends of the
lines that are drawn through the center of the epicycle, though they happen to be
determined by those lines. Accordingly the mean apogee of the epicycle is always in
the plane that cuts the plane of the deferent orthogonally in the line of the mean
apogee, and the true apogee of the epicycle is in a similar plane that cuts the defer-
ent in the line of the true apogee.

Fourth, it appears clearly that the centers of the deferents and equants deviate
from the plane of the ecliptic.

But the latitudes of these planets, which are written in the tables, refer to the case
when the center of the epicycle is at the point of the deferent with the greatest
inclination.

But Venus and Mercury usually have a threefold latitude: one arising from the
deferent, which is called the deviation; another arising from the inclination of the
diameter of the true apogee and perigee of the epicycle, which is called the inclina-
tion; and a third arising from the sloping of the diameter of the mean distances [of
the epicycle] with respect to the true apogee, which is called the slant.\[5.6v\]

For the plane of the deferent moves in latitude at one time to the north, at another
to the south, on a diameter of the world, and the poles of this motion are separated
on both sides of the apogee of the equant by ninety degrees of the ecliptic.\[5.7r\] for
the head and tail are there. This motion in latitude, however, is proportioned to the
motion of the center of the epicycle in such a way that when the center of the epi-
cycle is in one of the nodes, namely ninety degrees distant from the apogee of the
equant, the deviation of the deferent is zero, but its entire plane is in the plane of
the ecliptic. After that, when the center of the epicycle recedes from the node, the
deferent begins to deviate in such a way that the half which the center of the epicy-
cle is entering always deviates toward the north in the case of Venus and always
toward the south in the case of Mercury. The deviation is successively increased
until the center of the epicycle has arrived at the apogee or perigee of the deferent.
Then in fact the deviation is greatest: in the case of Venus ten minutes but in the
case of Mercury forty-five minutes;\[5.7r\] the deviation is continually reduced farther on
until the center of the epicycle has arrived at the other node. Here again the devia-
tion will become zero. Afterward it becomes again as before. Accordingly it ap-
pears, just as the center of the epicycle of Venus never deviates south from the
ecliptic, the center of the epicycle of Mercury never happens to deviate toward the
north. Again it is clear that the motion of circulation of the epicycle’s center in the
deferent is equal [in period] to the returning of the deferent in latitude. Hence simi-
larly it appears that the poles on which the motion of the deferent in longitude takes
place, as stated above, at one time approach the poles of the zodiac and at another
time recede from them.\[5.7r\] On account of the deviations just described, however, it

\[110\] Peurbach here introduces the terms deviatio (deviation), inclinatio (inclination), and reflectio
(slant) to describe the three angles respectively. On the latitude theory of the inferior planets see
R. C. Riddell, “The Latitudes of Venus and Mercury in the Almagest,” Archive for History of Exact

\[111\] This means that the apsidal line of the deferent is supposed to be perpendicular to the nodal line,
so that the apogee is at the top of the deferent. Whereas, however, the inclination of the plane of the
deferent to that of the ecliptic is constant in the case of the superior planets, in the case of Venus and
Mercury this inclination, here called the deviation, is variable, so that the plane of the deferent

\[112\] These values are taken from Ptolemy, Almagest 13.3, p. 601.

\[113\] The manuscript has the additional lines: “Quare etiam superficiem planam circuli quem centrum
seems necessary to add over and above the orbs already enumerated another concentric to the world and enclosing all those mentioned before, [an orb] in accordance with whose motion of trepidation the aforementioned deviations happen.

But the plane of the epicycle moves by inclination to the plane of the deferent on one side and the other. First, [it moves] about the diameter of the epicycle through the mean distances from the true apogee, and by this motion it happens that the diameter of the true apogee and perigee cuts the plane of the deferent in such a way that the true apogee inclines in one direction to the deferent and the perigee in the other. That inclination, however, is so proportioned to the motion of the center of the epicycle that whenever the center of the epicycle is in the apogee of the equant, the diameter of the true apogee and perigee in no place inclines to the deferent but is established in its plane. When the center of the epicycle is receding from the apogee of the equant, however, the true apogee of the epicycle begins to incline to the plane of the deferent—in the case of Venus toward the north, in the case of Mercury toward the south—and the true perigee begins to incline in the opposite direction. That inclination continually increases until the center of the epicycle has arrived at the node of the tail, namely, when it has departed from the apogee of the equant by ninety degrees eastward. For then the inclination of the diameter of the true apogee and perigee happens to be greatest. Thereafter it will continually diminish until the center of the epicycle has arrived at the perigee of the equant, when again this diameter nowhere inclines, but is established in the plane of the deferent. Thereupon, as the center of the epicycle recedes toward the other node, the true apogee begins to incline to the plane of the deferent—in the case of Venus to the south, but in the case of Mercury to the north—and the inclination is successively increased until the center of the epicycle has arrived at the other node, when again it will become a maximum. After that point, however, it decreases until it comes to the apogee of the equant, when, just as at the beginning, the said diameter will be in the plane of the deferent. Thereafter the former arrangement recurs. Therefore, whenever the deviation of the deferent happens to be greatest, the epicycle has no inclination, and when the former is zero, the latter is greatest.

Second, the plane of the epicycle moves from the plane of the deferent by inclining about the diameter of the epicycle passing through the true apogee and perigee. By this motion it happens that the diameter of the epicycle passing through the mean distances from the true apogee at some time cuts the plane of the deferent, in such a way that the left half of the epicycle is slanted from the deferent in one direction and the right half in the other. I call that left which is eastward from the apogee of the epicycle. However, this slant of the diameter is again so proportioned to the motion of the center of the epicycle that whenever the center of the epicycle is in the

deferentis Mercurii describit superficiem eclipticae secure ncesse est, superiore quidem mediate
eius ad meridiem, inferiore ad aquilonem declinante. Centrum tamen eius cum longitudinis medius
superficiei cohaerent" (Hence the plane of the circle described by the deferent of Mercury must
necessarily cut the plane of the ecliptic, its upper half in fact sloping to the south, the lower to the
north. However, its center with the mean distances agrees with the plane of the ecliptic.) Vienna
Codex 5203, fol. 20r.

114 Peurbach uses the term \textit{declinatio} here and subsequently instead of the term \textit{inclinatio}, which
he had carefully introduced for this angle at the beginning of his account of the latitude theory for
Venus and Mercury.

115 The motion of the diameter of the epicycle containing the true apogee and true perigee (\(d_t\)) is
thus similar to that in the case of the superior planets, except that the phase is different: for in the
case of Venus and Mercury, the inclination of \(d_t\) to the plane of the deferent is zero when the center
of the epicycle is in the apogee or perigee of the deferent and reaches its maximum value when the
center of the epicycle is in the nodes.

116 This is the diameter \(d_2\) perpendicular to \(d_1\). In the case of the superior planets, \(d_2\) remains
parallel to the plane of the ecliptic. In the case of Venus and Mercury, however, \(d_2\) has a variable
inclination (called the slant) to the plane of the ecliptic. The resulting oscillatory motion of \(d_2\) is
similar to that of \(d_1\) but with a different phase. Thus, as Peurbach goes on to explain, the slant is
greatest when the epicycle center is in the apogee or perigee of the deferent and zero when the
epicycle center is in the nodes (\textit{Almagest} 13.2, p. 599).
node of the head, namely, in the intersection ninety degrees west of the apogee of the deferent, the slant of the diameter of the mean distances is zero, but it is placed in the same plane with the deferent. But as the center of the epicycle from this point onward recedes toward the apogee, the left or eastern half of the diameter begins to be slanted to the plane of the deferent—in Venus to the north but in Mercury to the south—and the other half in the opposite direction. This slant continually increases until the center of the epicycle reaches the node of the tail, when the slant again becomes zero. But from this place, as the center of the epicycle passes toward the perige of the equant, the left half of the diameter passing through the mean distances again begins to be slanted—in Venus to the south but in Mercury to the north—and the slant will increase until the center of the epicycle comes to the perige of the equant, where it will then again become greatest. Thereafter, however, it successively decreases until the center of the epicycle returns to the node of the head, when the slant will become zero and the former pattern will again recur. Thereafter it is clear that, in the position of the deferent when the inclination of the epicycle reaches zero, its slant reaches the maximum. Thus the deviations are calculated from the ecliptic, but inclinations and slants from the deferent. And the values written in the tables are the ones that will occur when they [i.e., deviations, inclinations, and slants] become greatest. When, however, the maximum slant occurs, namely, when the center of the epicycle is in the apogee or perige of the deferent, the extremity of the diameter that is slanted has a smaller slant than many parts of the circumference of the epicycle that is located under the diameter toward the perige. However, the point of the circumference of the epicycle that touches the line drawn tangent to it from the center of the world then has the greatest slant of all [Figure 24]. Therefore just as the motion of inclination of the epicycle is made on the diameter that is slanted, so conversely the motion of slant of the epicycle takes place on the inclining diameter. Hence in turn, one is the axis of the motion of the other. Therefore it is not necessary in Venus and Mercury, as in the superior planets, that the axis on which the motion of inclination of the epicycle occurs, when it is outside the nodes, should be parallel to the plane of the ecliptic. On account of these inclinations and slants of the epicycles, some assume that small orbs have the epicycles within them, and that the same things happen to their motion.

"ON THE MOTION OF THE EIGHTH SPHERE"

A threefold motion belongs to the eighth sphere, by the motion of which, as has often been said, the orbs carrying the apogees of the planets are shifted. One comes from the first movable sphere, namely, the daily motion, by which it revolves about the poles of the world once in a natural day. Another comes from the ninth sphere, which is called the second movable and which always moves uniformly on the poles

\[5.8r\]

\[117\] Peurbach here (and also in the next sentence) uses the term *inclinatio*, which he defined at the beginning of the section.

\[118\] According to Nāṣir al-Din al-Ṭūsī (in his *Tadhkirah fi ʿilm al-hayāt*), such spheres were introduced by Alhazen, who thought two sufficient for each of the superior planets and four for each of the inferior planets; see the translation of Bernard Carra de Vaux in Paul Tannery, *Recherches sur l'histoire de l'astronomie ancienne* (Paris, 1893), pp. 337–361, esp. pp. 355–356. This explanation is not to be found in Alhazen’s astronomical treatise, *On the Configuration of the World* (*Kitab fi hayāt al-ʿalam*). Al-Ṭūsī improves Alhazen’s explanation using additional spheres. In the case of the superior planets two concentric spheres, enclosing the epicycle, produce the periodic deviation of the diameter \(d_1\) in accordance with the principle of the Tusi-couple, while a third concentric sphere surrounding these maintains the perpendicular diameter \(d_2\) in its position parallel to the plane of the ecliptic. For each inferior planet six spheres are required to produce the inclination of \(d_1\) and the slant of \(d_2\). Al-Ṭūsī thus explains the mechanism without violating the astronomical principle of uniform circular motion. A good account may be found in Willy Hartner, “Nāṣir al-Din al-Ṭūsī’s Lunar Theory,” *Physics*, 1969, 11:287–304, which also contains some criticisms of Carra de Vaux’s interpretations of al-Ṭūsī’s lunar theory.
of the zodiac eastward, against the first motion, so that every two hundred years it advances nearly one degree and twenty-eight minutes. In the tables this is called the [mean] motion of the apogees and the fixed stars. And it is the arc of the zodiac of the first movable between the beginning of Aries of the first movable and the beginning of Aries of the ninth sphere. For the plane of the ecliptic of the ninth sphere is always in the plane of the ecliptic of the first movable. The third motion, however, which is called the motion of trepidation or approach and recession of the eighth sphere, is special to this sphere and is made on two small circles in the concavity of the ninth sphere. These two circles are described equally over the beginnings of Aries and of Libra of the same sphere, in such a way that two particular points of the eighth sphere, diametrically opposite, which are called the beginnings of Aries and of Libra of the eighth sphere, uniformly describe the circumferences of those two circles of the ninth sphere [Figure 25]. In addition, the ecliptic of the eighth sphere intersects the ecliptic of the ninth—at least when it intersects—in the diametrically opposite beginnings of Cancer and Capricorn of the ninth sphere. Thence it follows that when one of these points of the eighth sphere is

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119 This is the rate of precession implied in the Alfonsine Tables, i.e., a complete revolution in 49,000 years.
120 In this context, caput is translated as “beginning.” Peurbach sometimes uses the term princi-pium.
121 The beginning of Aries of the first movable is the position that would be occupied by the equinox in the absence of precession. As the ninth sphere rotates about the poles of the ecliptic, it carries the equinox (the beginning of Aries of the ninth sphere) along the ecliptic of the first movable. Moreover, as the ecliptic of the ninth sphere is supposed fixed, it is in the same plane as the ecliptic of the first movable.
122 The theory of the motion of the eighth sphere described by Peurbach, involving a combination of precession and trepidation, was introduced by al-Battani and al-Farghani. It is the theory on which the Alfonsine Tables are based.
123 In this diagram the small circles must be interpreted as being perpendicular to the paper and the straight lines joining points on them as great circles of the sphere.
124 This is clearly movable, since that of the ninth sphere, on which are located the centers of the small circles, is fixed.
125 The argument is easier to follow using the perspective Fig. 26A. EE represents the equator, nn the ecliptic of the ninth sphere, and ee the ecliptic of the eighth sphere. The beginnings of Aries and Libra of the ninth sphere are represented by a and b (the centers of the small circles) and the corresponding points of the eighth sphere by A and B. The equinoxes are represented by L and M. P and Q, the points of intersection of the two ecliptics (defined as the beginnings of Cancer and Capricorn of the ninth sphere), are at a distance of 90°, measured along the ecliptic, from a and b.
in the southern half of its circle, the other will be in the northern half of its circle. Also the ecliptic of the eighth sphere always cuts the ecliptic of the ninth, when it intersects it, in equal parts, and again the parts of the small circle in turn are equal.

The rule of the speed of this motion in fact is as follows: either one of the two points completes the circumference of the small circle in which it is carried around in seven thousand years, in round numbers.\textsuperscript{126} However, although by this motion the two points mentioned above, namely, the beginnings of Aries and Libra of the eighth sphere, describe two equal circumferences of circles, no other points on the eighth sphere happen to describe circumferences of circles. In fact it is necessary that the beginnings of Cancer and Capricorn of the eighth sphere complete as it were conical figures that have for their bases curved lines on both sides from the beginnings of Cancer and Capricorn of the ninth.\textsuperscript{127} Accordingly the beginnings of Cancer and Capricorn of the eighth sphere will sometimes precede the beginnings of Cancer and Capricorn of the ninth sphere and sometimes follow; sometimes, however, they are united. For the beginning of Cancer of the eighth and the beginning of Cancer of the ninth are united when the beginning of Aries of the eighth is at its greatest latitude from the ecliptic of the ninth, which [latitude] occurs in the great circle passing through the poles of the zodiac of the ninth and the centers of the circles.\textsuperscript{128}

And the poles of the ecliptic of the eighth, improperly called poles, sometimes approach the poles of the ecliptic of the ninth, sometimes are under them, and sometimes recede from them. Such approach and recession, however, are always along the great circle passing through the poles of the zodiac of the ninth and the centers of the small circles.\textsuperscript{129} It happens, therefore, that the ecliptic of the eighth sphere, as its situation varies, intersects successively in various of its parts the equator of the first movable. Such an intersection also occurs now in the actual beginning of Aries of the first movable, now on the near side, now on the far side, so that, in the time taken by the center of the small circle to complete one revolution, which in everyday speech happens in forty-nine thousand years, any point whatever of the ecliptic of the eighth sphere will cut the equator near the beginning of Aries and also again near the beginning of Libra of the first movable. These intersections in the equator, in fact, sometimes seem to approach the beginnings of Aries and Libra of the first movable, but sometimes seem to move away from them, advancing at one time eastward and at another westward. Consequently it happens that the greatest declinations of the zodiac are variable. It is for this reason, it is believed, that different astronomers in different periods happen not to have found the same quantities for these maximum declinations of the zodiac. For Ptolemy found them to be greater than Almeon\textsuperscript{130} did. Since anyhow they advanced with similar ways and means, that could hardly happen otherwise than by diversity of motion of the sort mentioned here, or similar means. But the variations in the intersection of the ecliptic of the eighth sphere and of the equator of the first movable with respect to Aries necessarily entail that the equinoxes, and similarly the solstices, vary continually. Consequently it is not necessary for the equinox always to occur when the sun is in the beginning of Aries of the first movable, but it may have been before or be about to follow after, namely, when the sun is in the aforementioned intersection.

Since indeed, as has been said above, the orbs carrying the apogee of the sun move about the axis of the ecliptic of the eighth sphere, according to the motion of the same sphere, and the orb carrying the sun moves about an axis parallel to that axis, it will necessarily follow that the center of the body of the sun is always found

\textsuperscript{126} This again is the period of trepidation implied in the Alfonsine Tables.

\textsuperscript{127} See Fig. 25 and the interpretation in n. 123.

\textsuperscript{128} In this case the fixed beginning of Cancer of the ninth sphere coincides with the variable beginning of Cancer of the eighth sphere at the point of intersection of the two ecliptics.

\textsuperscript{129} The plane of the movable ecliptic of the eighth sphere oscillates about the fixed diameter through the beginnings of Cancer and Capricorn of the ninth sphere.

\textsuperscript{130} I.e., al-Ma'dmün, Caliph of Baghdad (786–833). His astronomer, Jahjá ibn Abi Mansūr, Abū cAli (see Suter, Die Mathematiker [cit. n. 71], p. 8) found the obliquity of the ecliptic to be $23^\circ$ 35'; see Al-Bāṭānī, ed. Nallino (cit. n. 97), p. 157. Ptolemy found the obliquity to be $23^\circ$ 51’.
in the plane of the ecliptic of the eighth sphere. But this plane is often or rather frequently outside the beginning of Aries of the first movable; for this reason what has been proposed above follows. The argument concerning the variation of the solstices is similar. From these things it is concluded, first, that it is not necessary for the sun, when in the beginning of Aries or Libra of the first movable, to have no declination from the equator. Second, it is likewise not necessary for the sun, when in the beginning of Cancer or Capricorn of the first movable, to have maximum declination from the equator, because it is possible for the sun to be in the circle passing through the poles of the ecliptic of the first movable and the beginning of Aries of the same, and nevertheless to be outside the plane of the equator. Similarly it is possible for the sun to be in the circle passing through the poles of the zodiac of the first movable and the beginning of Cancer of the same, and nevertheless not then to have maximum declination from the equator, but to have been in that condition before, or about to be in it after.

This also follows; that the tropics [Figure 27] of Cancer and Capricorn vary continuously with respect to the equator, now approaching it, now receding from it, but that variation has certain limits, which it cannot overstep.

From these considerations, it is sufficiently clear that to the motions of the stars is added motion from the motions of the ninth sphere and trepidation of the eighth sphere, sometimes eastward, now fast, now slow, sometimes stationary, and sometimes westward, following the variation of the situation of the beginning of Aries of the eighth sphere in the circumference of its small circle. Therefore it was very difficult for the ancients to discover the quality of this motion. Consequently different notions have been variously imagined in this matter. For some said that the apogees and fixed stars moved through nine hundred years continuously toward the east up to seven degrees, then through another nine hundred years just as much in the opposite direction toward the west.132 But Albatagni said that they moved one

131 The arc through the centers of the small circles represents the equator and the double circle one of the tropics. The lines joining the tropic to the equator show the limits of the solstice.

132 Peurbach here follows the Theorica planetarum (Theorica Gerardi, ed. Carmody [cit. n. 77], p. 46). Theon of Alexandria relates that some ancient astrologers supposed such a periodic motion of the equinoxes with a limit of 8° eastward and westward and a rate of 1° in 80 years. According to this supposition, the equinoxes would move eastward for 640 years and then westward for the same interval of time. See Al-Bāṭṭānī, ed. Nallino (cit. n. 97), p. 126; cf. Neugebauer, History (cit. n. 89), pp. 631–632.

133 I.e., al-Bāṭṭānī.
degree always toward the east in sixty years and four months.\textsuperscript{134} Alfraganus,\textsuperscript{135} however, supposed that in one hundred years they would complete one degree, always toward the east.

Therefore the mean motion of approach and recession of the eighth sphere [Figure 28\textsuperscript{136}] is the arc of the small circle reckoned from the topmost point of the quadrant eastward up to the beginning of Aries of the eighth sphere.

But the equation of the eighth sphere is the arc of the ecliptic of the ninth sphere intercepted between the center of the small circle and the great circle from the poles of the ecliptic of the ninth passing through the beginning of Aries of the eighth. When therefore the mean motion of approach and recession is zero or the semicircle, the equation of the eighth sphere becomes zero. But if it is 90 or 270 degrees, it will be maximum. When however such motion of approach and recession is less [6.3r] than a semicircle, the equation should always be added; but when it is more, the equation should be subtracted.

Thabit\textsuperscript{137} says that only two motions belong to the eighth sphere: one from the first movable or ninth sphere,\textsuperscript{138} namely, the diurnal, the other of its own, that is to say, of the trepidation, which it makes about the small circles.\textsuperscript{139} He asserts the ecliptic to be of two kinds, fixed in the ninth sphere but movable in the eighth, so

\textsuperscript{134} Peurbach again relies on the \textit{Theorica planetarum} (\textit{Theorica Gerardi}, ed. Carmody [cit. n. 77], p. 46) for this false statement. Al-Būtāni in fact found the rate of precession to be 1° in 66 years; \textit{Al-Būtāni}, ed. Nallino, (cit. n. 97), p. 124.

\textsuperscript{135} I.e., al-Farghani; see Suter, \textit{Die Mathematiker} (cit. n. 71), pp. 18–19. The rate of precession of 1° in 100 years was the value adopted by Ptolemy (\textit{Almagest} 7.2, p. 328).

\textsuperscript{136} This is easier to understand if the points of the diagram are labeled, as in the redrawn Fig. 28A. Z is the pole of the ecliptic of the ninth sphere. A is the beginning of Aries of the first movable. B is the beginning of Aries of the ninth sphere. The beginning of Aries of the eighth sphere moves around the small circle. When the beginning of Aries of the eighth sphere has moved from F to G, the mean motion of approach and recession is the arc FG and the equation of the eighth sphere is the arc BD of the ecliptic of the ninth.


\textsuperscript{138} Note that this is not the ninth sphere of the previous theory.

\textsuperscript{139} Thus Thābit supposes that the phenomenon of the variation of the equinoxes can be completely described in terms of trepidation.
that the beginnings of the movable Aries and Libra are carried around in two small
circles [Figure 29], the centers or poles of which are themselves the beginnings of
Aries and Libra of the fixed ecliptic, and the arc of the fixed ecliptic between the
poles\(^{140}\) of those small circles and their circumferences contains four degrees, eight-
teen minutes, and forty-three seconds.\(^{141}\)

He says again\(^{142}\) that the beginnings of the movable Aries and Libra are carried
around in such a way that, when the beginning of the movable Aries is in the west-
ern intersection of the small circle and the equator, it will itself move into the half of
the small circle that is north of the equator, but the beginning of the movable Libra
then moves through the half of its small circle that is south of the equator. And when
the beginning of the movable Aries is in the eastern intersection of the equator and
its small circle, it will move into the half of the small circle that is south of the
equator. But the beginning of the movable Libra then revolves through the half of its
small circle north of the equator. Moreover, when the beginning of the movable
Aries is in either of the two points of intersection of the fixed ecliptic with the small
circle, the movable ecliptic will be placed directly in the plane of the fixed ecliptic,
which will happen twice in one revolution of the beginning of the movable Aries in
its small circle. But when the beginning of the movable Aries is located in all other
positions of the periphery of its small circle, the movable ecliptic will cut the fixed
ecliptic in points at the beginning of the movable Cancer and Capricorn.\(^{143}\) For these
two points of the movable ecliptic always cling to the circumference of the fixed
ecliptic in this motion, so that they never recede from it. But it happens that
they separate from the beginnings of the fixed Cancer and Capricorn by a quantity

\(^{140}\) I.e., the centers.

\(^{141}\) I.e., the radii of the small circles subtend angles of 4° 18' 43'' on the celestial sphere; see

\(^{142}\) The argument is easier to follow using the perspective diagram Fig. 26A. As before, EE repre-
sents the equator, but now nn represents the fixed ecliptic and ee the movable ecliptic. P and Q, the
points of intersection of the two ecliptics (defined as the beginnings of the movable Cancer and
Capricorn), are at a distance of 90°, measured along the movable ecliptic, from A and B.

\(^{143}\) Three points are needed to define the position of the movable ecliptic. The diametrically oppo-
site points on the circumferences of the two small circles give two of these. Placing the beginning of
Cancer of the movable ecliptic on the fixed ecliptic completes the determination. Thus Thabit's
theory is slightly different from the theory of trepidation described earlier, in which the position of
the movable ecliptic was determined by defining the point of intersection as the beginning of Cancer
of the fixed ecliptic.
of four degrees, eighteen minutes, and forty-three seconds toward east or west.

Moreover, wherever the intersection of these ecliptics is made, it must be sepa-
rated from the beginnings of the movable Aries and Libra by a quadrant of a great
circle. Although, in fact, in one revolution of the beginning of the movable Aries in
its small circle, it happens twice that the beginnings of the movable Cancer and
[6.3v] Capricorn are located under the beginnings of the fixed Cancer and Capricorn, the
beginnings of the movable Aries and Libra never arrive under the beginnings of the
fixed Aries and Libra. For when the movable ecliptic touches the small circle on the
northern side in the point of the movable Aries, the movable beginnings of Cancer
and Capricorn are joined with the beginnings of the fixed. It happens in like manner
in the southern contact. But the beginnings of Aries and Libra are always separated
from the beginnings of the fixed by the quantity that has been stated [at the end of
the paragraph above]. Moreover, the fixed ecliptic always cuts the equator in the
beginnings of the fixed Aries and Libra and always at the same angle, namely,
twenty-three degrees, thirty-three minutes, and thirty seconds. But the movable
ecliptic cuts the equator successively in particular points included in the two arcs,
which the movable ecliptic cuts off from the equator in the two positions of con-
tact, and the quantity of each is about twenty-one degrees and thirty minutes. For
the maximum distance of the beginning of the movable Aries from the intersections
of the [movable] ecliptic with the equator is ten degrees and forty-five minutes.
Accordingly the maximum declination of the movable ecliptic from the equator is
variable: sometimes more than the inclination of the fixed ecliptic, sometimes less
than the same, and sometimes equal to it. For the one is equal to the other when the
movable is under the plane of the fixed, and more when they are in the positions of
contact. Consequently Ptolemy found it to be thirty-three degrees, fifty-one min-
utes, and twenty seconds. But it is smaller when the beginning of the movable
Aries is in the intersection of the equator and the small circle, for then the intersec-
tion of the ecliptics will be in the point of the movable ecliptic with greatest declina-
tion, which declines less than the fixed beginning of Cancer and Capricorn.
Therefore the equation of the eighth sphere is the arc of the movable ecliptic
intercepted between the beginning of the movable Aries and the intersection of the
same ecliptic with the equator. But the motion of approach and recession is the arc
of the small circle between the beginning of the movable Aries and the intersection
of the equator and small circle, advancing through the northern half of the circle.
By this motion it happens that the fixed stars appear to move now toward the
east, now toward the west, now with a fast motion, now with a slow motion. For
when the beginning of the movable Aries is in the quadrants of the small circle
[going] from the equator, that is to say, near the positions of contact about which we

144 According to Thābit, the obliquity of the fixed ecliptic was found by the observers of the time of
al-Ma’mun to be 23° 33'; Neugebauer, "Thābit" (cit. n. 137), p. 293. The value quoted by Peurbach
(or Regiomontanus) is that of al-Zarqālī; Ernst Zinner, Leben und Wirken des Johannes Müller von
145 i.e., when the movable ecliptic touches the small circles.
146 In this case the movable and fixed ecliptics intersect in the beginnings of Cancer of the fixed,
i.e., the point of greatest declination of the fixed ecliptic. Since this is not also the point of maximum
declination of the movable eccentric, it follows that the inclination of the movable ecliptic to the
equator is greater than that of the fixed ecliptic. It should be noted, as Peurbach (or Regiomontanus)
explains later, that the points of maximum declination of the movable ecliptic (i.e., the solstices) are
not necessarily in the beginnings of Cancer and Capricorn of this ecliptic. For the beginnings of the
movable Cancer and Capricorn are separated by a quadrant from the points of the movable begin-
nings of Aries and Libra on the small circles, whereas the points of maximum declination are sepa-
rated by a quadrant from the intersections of the movable ecliptic and the equator.
147 This is a misprint for 23° 51' 20".
148 In this case the points of intersection of the movable ecliptic with the equator coincide with the
movable beginnings of Aries and Libra on the small circles. It follows that the points of maximum
declination of the movable ecliptic coincide with the movable beginnings of Cancer and Capricorn.
Since these are the points of intersection of the two ecliptics, and the fixed Cancer and Capricorn, at
which the declination of the fixed ecliptic is greatest, are elsewhere, it follows that the maximum
declination of the movable ecliptic is less than that of the fixed ecliptic.
have spoken, the fixed stars seem to move slowly in that direction toward which they are moving, because then the equation of the eighth sphere increases or decreases little. But when the beginning of the movable Aries is in or near one or other of the intersections of the equator and the small circle, the stars will seem to move quickly in that direction toward which they are moving, because under these conditions, the equation of the eighth sphere increases or decreases very much.

As a result the evident diversity in their motion has been found. For Ptolemy compared their positions, verified for his time, to their positions discovered by Hipparchus and others, and found that they had moved with a slow motion, that is to say, by one degree in one hundred years. For then the beginning of Aries had departed from the point of the southern quadrant of the small circle, approaching toward the equator. His successors, however, when it was approaching closer, found that it was moving one degree in sixty-six years. Now in our time, that is to say, 1460 A.D., the beginning of Aries has come to be in the north, almost sixty-six degrees distant from the intersection of the small circle and the equator. Hence it is distant from the intersection of the movable ecliptic with the equator almost nine degrees, forty-eight minutes. Therefore the intersection now occurs on twenty degrees, twelve minutes of Pisces of the movable ecliptic.

But the maximum equation of the eighth sphere happens when the beginning of the movable Aries is on the points that mark off the quadrants of the small circle from its intersections with the equator, and is ten degrees, forty-five minutes. Accordingly, any point of the movable ecliptic from nineteen degrees, fifteen minutes, of Pisces up to ten degrees, forty-five minutes, of Aries can be in the position of the intersection that is the point of the vernal equinox. The same is to be understood concerning the point of the autumnal equinox in the opposite arc. It is clear also that the points of the solstices\textsuperscript{49} are not always in the beginnings of the movable Cancer or Capricorn, but in points separated by a quadrant from the intersection of the equator with the movable ecliptic. Ptolemy therefore, judging the stars in his time to move from west to east, believed that there was only one fixed zodiac, that is to say, one that would always have the same inclination to the equator, and what he said follows in relation to that belief. For, since the southern stars approached toward the point of the vernal equinox when they receded from the winter solstice, and receded northward from the equator when they appeared between this point and the summer solstice, he judged them to move eastward. But [Thābit’s] motion being assumed in [Ptolemy’s] time, the southern stars were in reality moving westward on the fixed ecliptic. Nevertheless it is true that, because the equation of the eighth sphere then decreased, the southern stars appeared to be moving eastward, since he supposed that the beginning of Aries of the fixed zodiac was in the intersection of the movable ecliptic with the equator, which intersection, he deemed, was always fixed.

All the inferior spheres in their motions follow this motion [of the eighth sphere], in such a way that, with respect to the movable ecliptic [of the eighth sphere], the apogees of the deferents and their declinations are always invariable.

\textsuperscript{49} puncta tropica.