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Source: *Isis*, Vol. 59, No. 4 (Winter, 1968), pp. 384-401
Published by: The University of Chicago Press on behalf of The History of Science Society
Stable URL: http://www.jstor.org/stable/228489
Accessed: 12/01/2009 11:02

The Enigma of Domingo de Soto: *Uniformiter difformis* and Falling Bodies in Late Medieval Physics

By *William A. Wallace, O.P.**

THE AIM of this discussion is to cast light on what Alexandre Koyré has referred to as "the enigma of Domingo de Soto."[1] Soto was a Spanish Dominican who, in the early sixteenth century, studied at the University of Paris, returned to Spain, and at the University of Salamanca composed a commentary and questions on the *Physics* of Aristotle (*c.* 1545) along with an imposing series of works on political philosophy and theology.[2] In a much-quoted passage in his questions on the *Physics* Soto associates the concept of motion which is *uniformiter difformis*—an expression deriving from the English *Calculatores*—with falling bodies, and he indicates that the distance of fall can be calculated from the elapsed time by means of the so-called Mertonian mean-speed theorem.[3] The casual way in which Soto introduces this association has led some to speculate that this was generally known in his day and that he merely recorded what had become common teaching in the early sixteenth century. "But if this is the case," writes Koyré, "why was de Soto alone in putting [these views] down on paper? And why did no one else before

* The Catholic University of America, Washington, D.C. Work for this paper was supported by a grant (GS-862) from the National Science Foundation, whose help is gratefully acknowledged.

[1] In his essay on science in the Renaissance in René Taton (ed.), *History of Science,* Vol. II: *The Beginnings of Modern Science, from 1450 to 1800,* trans. A. J. Pomerans (New York: Basic Books, 1964), pp. 94–95.

[2] The best documentary study of Soto's life and works is Vicente Beltrán de Heredia, O.P., *Domingo de Soto: Estudio biográfico documentado* (Salamanca: Convento de San Esteban, 1960).

[3] As will be explained, the application of the expression *uniformiter difformis* to falling bodies is equivalent to stating that the motion of such bodies is uniformly accelerated. The method of calculating the distance traversed in such a motion was first worked out at Merton College, Oxford, by a group of scholars usually referred to as the Mertonians, or *Calculatores.* Their method consisted in replacing the uniformly varying speed by an average or mean value and then using this to compute the distance of travel; the equivalence on which this technique was based has come to be known as the "mean-speed theorem." Actually the method was applied in a broader context to all types of change, whether quantitative or qualitative, and the theorem could be referred to more generally as the "mean-degree theorem." Likewise the expression *uniformiter difformis* was applied to all kinds of change, but in what follows we shall restrict ourselves exclusively to applications to local motion and thus omit any discussion of the extensive literature that developed relative to other applications.

Galileo . . . adopt them?[4] These questions, neither of which is easily answered, pose the enigma of Domingo de Soto.

The answer to the riddle should be forthcoming from a study of the teachings of Soto's predecessors and contemporaries, and it is such a study that provides the background for the present paper. The results show that Soto is still probably the first to associate the expression *uniformiter difformis* (with respect to time) with the motion of falling bodies. The association itself, however, is not completely fortuitous: it appears to be the result of a progression of schemata and exemplifications used in the teaching of physics from the fourteenth to the sixteenth centuries. I intend to trace this development and to show the extent to which Soto's presentation of the material was novel. Soto's uniqueness, it seems to me, consists in having introduced as an intuitive example the simplification that Galileo and his successors were later to formulate as the law of falling bodies. How Soto came to his result appears to be a good illustration of the devious route that scientific creativity can follow before it terminates in a new theoretical insight that is capable of experimental test.

For the sake of convenience I shall divide the presentation on the basis of its approximate chronology, treating successively those schemata and exemplifications used in discussing local motion in the fourteenth, fifteenth, and sixteenth centuries. The background is, of course, Aristotelian, but in the foreground are to be found the various Mertonian and nominalist distinctions that figured prominently in the emerging science of mechanics.

Fourteenth Century

In the first schema to be discussed the basic distinctions were foreshadowed in the works of Gerard of Brussels and of Thomas Bradwardine, but they came to be known by the mid-fourteenth century through the writings of Bradwardine's disciples William Heytesbury and Richard Swineshead.[5] These distinctions were applied generally to the intensification of changes or motions; as applied to local motion "intension" became synonymous with velocity or its change, and thus various qualifying adjectives such as "uniform" and "difform" came to have kinematical significance. A uniform motion U is one with constant velocity v, whereas a difform motion D has a changing velocity. Further, a motion may be uniform in either of two senses: with respect to the parts of the object moved, symbolized $U(x)$, in the sense that all parts of the object move with the same velocity; or with respect to time, symbolized $U(t)$, in the sense that the velocity of the object as a whole remains constant over a time interval. This distinction may also be applied to difform motions, yielding the two corresponding types, $D(x)$ and $D(t)$. With difformity, moreover, a further series of distinctions may be introduced. Motion that is difform with respect to the parts of the object moved may be either uniformly difform, $UD(x)$, in the sense that there is a uniform (spatial) variation in the velocity of the various parts of the object, or difformly difform, $DD(x)$, in the sense that there is no such uniform (spatial) variation. Again, motion may be uniformly difform with respect to time, $UD(t)$, or difformly difform in the same sense, $DD(t)$. Both of these in turn may be subdivided on the basis of the direction of the change—that is, whether it is increasing or decreasing—to yield uniformly accelerated motion, $UD_{acc}(t)$, and uniformly decelerated motion, $UD_{dec}(t)$, or alternatively,

[4] Koyré, in Taton, *op. cit.*, p. 95.

[5] For a general survey of this development see Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison: Univ. Wisconsin Press, 1959).

difformly accelerated motion, $DD_{acc}(t)$, and difformly decelerated motion, $DD_{dec}(t)$. The resulting eight possibilities, all of which are capable of exemplification, are shown in Schema I; the symbols on the right will serve to number the particular types of motion in the schema and the examples that were proposed to concretize their definitions.

SCHEMA I

v	U	$U(x)$			I-1
		$U(t)$			I-2
	D	$D(x)$	$UD(x)$		I-3
			$DD(x)$		I-4
		$D(t)$	$UD(t)$	$UD_{acc}(t)$	I-5
				$UD_{dec}(t)$	I-6
			$DD(t)$	$DD_{acc}(t)$	I-7
				$DD_{dec}(t)$	I-8

The complete articulation of all the subdivisions of Schema I was not given, to my knowledge, by any author before Soto, although the main lines of the division were already implicit in Heytesbury (1335). All of the English *Calculatores*, however—and in this designation I include Heytesbury as well as Swineshead (*c.* 1340), the English logician Robert Feribrigge (*c.* 1367), and the pseudo-Bradwardine (the author of the *Summulus de motu,* published in 1505)[6]—were content to define the various kinds of motion in abstract and mathematical terms, without illustrations from the physical universe. However, on the Continent, at the University of Prague, John of Holland (*c.* 1369)[7] repeated most of the divisions in Schema I and when explaining his definitions provided four examples: motion of type I-1 he illustrated with a falling stone (*motus lapis deorsum*), all of whose parts move at the same speed; type I-2, with an object (*mobile*) moving in uniform translation, or, alternatively, with a sphere (*spera*) in uniform rotation; type I-3, with the motion of the ninth sphere (*nona spera*), some of whose parts move more slowly than others even though the whole rotates uniformly; and type I-5, with the example of Socrates (*Sortes*) continually accelerating his walking speed. In passing, when defining types I-5 and I-6, John of Holland referred to the definitions given by the *Calculatores*, a reference that serves to align him with the Mertonian tradition. His work, to my knowledge, provides the fullest exemplification of Schema I of those written in the fourteenth century.

Writing at about the same time as John of Holland but at the University of Paris, Albert of Saxony[8] and (somewhat earlier) Nicole Oresme[9] utilize another

[6] On Robert Feribrigge see Clagett, *ibid.*, pp. 630–631; the text of the *Summulus de motu* is given by Clagett also, on pp. 445–462.

[7] For the relevant texts, *ibid.*, pp. 247–250.

[8] Albert of Saxony, *Questiones super quatuor libros Aristotelis de celo et mundo. . .* (Venice: 1520).

[9] See the excerpt from Oresme's *On the Configuration of Qualities* in Clagett, *Mechanics in the Middle Ages,* pp. 347–381. For the dating of Oresme's scientific writings see Clagett's article on Oresme in the forthcoming *Dictionary of Scientific Biography* (Scribner's); this has been pre-printed by the American Council of Learned Societies (New York, 1967) as a sample article for the guidance of authors preparing articles for the *Dictionary.*

way of classifying types of local motion. Both refer to motions being uniform or difform according to parts, $U(x)$ and $D(x)$, and according to time, $U(t)$ and $D(t)$, although Albert prefers to speak of the latter motions as "regular" and "irregular."[10] Again, both are concerned to supply examples and in so doing group the types of motion in a way that influenced later writers. Rather than consider one independent variable at a time, they take two variables together and speak of motion being, for example, "uniform and regular," which may be symbolized as $U(x) \cdot U(t)$, or "uniform and irregular," symbolized as $U(x) \cdot D(t)$, or "difform and regular," $D(x) \cdot U(t)$. The four possibilities that result from this classification are given in Schema II.

Schema II

$$v \begin{cases} U(x) \cdot U(t) \ldots\ldots\ldots\ldots\ldots\ldots \text{II-1} \\ U(x) \cdot D(t) \ldots\ldots\ldots\ldots\ldots\ldots \text{II-2} \\ D(x) \cdot U(t) \ldots\ldots\ldots\ldots\ldots\ldots \text{II-3} \\ D(x) \cdot D(t) \ldots\ldots\ldots\ldots\ldots\ldots \text{II-4} \end{cases}$$

The examples provided by Albert and Oresme are particularly interesting in that a falling body is used as an illustration of local motion. Thus Albert's example of type II-2 is a heavy object (*grave*) or a falling stone (*lapis*): in the latter case the motion is uniform, $U(x)$, because all parts of the stone move with the same velocity at any instant, but irregular, $D(t)$, because the stone moves faster "at the end than at the beginning."[11] His example of type II-3, on the other hand, is a wheel (*rota*) whose motion is difform, $D(x)$, because "the parts close to the axle do not move as fast as those close to the circumference," but regular, $U(t)$, because the angular velocity of the whole remains constant.[12] A third example, type II-1, is described by Albert as follows:

> Similarly note a third possibility, that there is no difficulty in a motion being both uniform and regular at the same time: for when a heavy object descends in a medium

[10] The Latin text reads as follows (*Questiones*, lib. 2, quest. 13):

> Regularitas autem motus attenditur ex parte temporis, ita quod motus ille dicitur regularis quando ipsum mobile movetur eque velociter in una parte temporis sicut in alia. . . . Verumtamen sciendum est quod aliqui distinguunt de uniformitate motus dicentes quod potest attendi vel ex parte partium mobilis, vel ex parte partium temporis. Uniformitas primo modo dicta est omnino eadem cum uniformitate distincta contra regularitatem; sed uniformitas secundo modo dicta est eadem cum regularitate. Sed illi non utuntur ita proprie uniformitate sicut nos utimur secundum dictas descriptiones.

[11] A fuller excerpt from the Latin text (*ibid.*):

> Ulterius sciendum est quod non est inconveniens aliquem motum esse uniformem et non esse regularem; patet de motu gravis deorsum in medio uniformi quod movetur uniformiter, quia una eius pars movetur ita velociter sicut alia, et tamen non movetur regulariter, quia movetur in fine velocius quam in principio.

Earlier in this question Albert gave the same example:

> . . . sicut si lapis aliquis descenderet, non obstante quod ille motus in fine esset velocior quam in principio; tamen diceretur uniformis secundum propriam significationem vocabuli ex eo quod una medietas illius lapidis descenderet ita velocior sicut alia.

[12] *Ibid.*:

> Similiter non est inconveniens aliquem motum esse regularem et tamen non esse uniformem; patet de rota que in equalibus partibus temporis equalia spatia describeret, talis enim motus rote esset regularis, non tamen uniformis, postquam [sic] partes illius rote centrales non moverentur ita velociter sicut partes circumferentiales.

> whose resistance is so regulated that the heavy object covers equal distances in equal parts of time, the motion of the heavy object would be both uniform and regular.[13]

This example is peculiar in that it is more complicated than it need be: a stone in uniform translational motion would satisfy the case, as it did for John of Holland. Albert's example, however, is consistent with the discussions of the English *Calculatores* relating to motion through various resistive media and was perhaps suggested by them.

In the texts I have analyzed, Albert gives no example of type II-4—difform and irregular motion. Nicole Oresme, however, does exemplify all four types for Schema II, although he is cryptic in doing so. In place of the wheel for type II-3 he mentions the movement of the heavens (*celum*) as being difform and regular. Then he goes on: "Conversely, the movement downward of a heavy body can be uniform and irregular [II-2], and it can be also uniform and regular [II-1], or even difform and irregular [II-4]."[14] Here Oresme makes the falling body cover all three remaining possibilities, although he gives no indication as to what modalities must be superimposed on its motion in order to satisfy the various definitions.[15]

To provide background for an interpretation of authors to be considered later in this paper, it will be convenient to note the mathematical descriptions of falling motion that were proposed by Albert and Oresme. In their commentaries on Book II of Aristotle's *De caelo et mundo* both discuss a variety of possibilities but do not use the terms *uniformiter, difformiter,* or *uniformiter difformiter*. Oresme mentions that the velocity of fall either increases with time arithmetically toward infinity or else increases with time convergently (as do proportional parts) toward a fixed limit; he elects for the first possibility. Albert also mentions these two possibilities and elects similarly, although in his first mention he is ambiguous as to whether he regards the velocity as varying linearly with the time of fall or with the distance of fall. Later he is more explicit and mentions three additional possibilities: (1) the velocity receives equal increments in the proportional parts of time (thus going to infinity exponentially within a finite time); (2) the velocity receives equal

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[13] *Ibid.*: Similiter sciendum est tertio non esse inconveniens aliquem motum simul esse uniformem et regularem, sicut si aliquod grave descenderet in aliquo medio sic proportionato ex parte resistentiae, quod illud grave in equalibus partibus temporis equalia pertransiret spatia; tunc enim motus illius gravis esset uniformis et regularis simul.

[14] The Latin text is provided by Clagett, *Mechanics in the Middle Ages*, p. 375:

> Motus vero gravis deorsum potest esse econverso uniformis et irregularis et etiam potest esse uniformis et regularis, vel difformis et irregularis. Sed non est possibile quod motus circularis sit uniformis. Verumptamen in sequendo modum loquendi consuetum vocabo quamcunque regularitatem nomine uniformitatis et irregularitatem nomine difformitatis. . . .

In his English translation of this, which differs from the one I provide, Clagett alters the punctuation and inserts an ellipsis that seems to me to change the sense. He translates: "Conversely, the movement downward of a heavy body can be 'uniform,' and 'irregular' or 'regular'" (p. 355). Prof. Clagett informs me that he has a new text with a correct translation in press, in a work titled "Nicole Oresme and the Medieval Geometry of Qualities and Motion."

[15] "Uniform and irregular" is obviously the case of a body falling freely in a uniform medium. "Uniform and regular" could be the case of a body falling in a medium whose resistance continually increases so as to prevent the possibility of a velocity increase (as in Albert of Saxony's example); whereas "difform and irregular" could be the case of a body falling freely in a uniform medium and rotating as it falls.

increments in the proportional parts of the distance traversed (thus going to infinity exponentially within a finite distance); (3) the velocity receives equal increments in equal parts of the space traversed (thus going to infinity linearly as a function of distance). Albert here elects the third possibility, showing that his earlier ambiguity should be resolved in favor of a spatial rather than a temporal variation. Clagett has analyzed all of these texts to show that none explicitly identifies falling motion as uniformly difform in such a way as to allow the Mertonian "mean-speed theorem" to be applied to it in unequivocal fashion.[16]

Schema II, it should be noted, did not enjoy the same popularity among later writers as did Schema I. However, types of motion that would fit into one or another of its categories were mentioned by the pseudo-Bradwardine, by Gaetano da Thiene, and by John Dullaert of Ghent. The principal importance of Schema II is that it introduced the two-variable concept and that it became the major vehicle for presenting falling bodies as exemplifications of the types of velocity variation in local motion being discussed in the late Middle Ages.

Fifteenth Century

Moving now to the fifteenth century, we come to a series of Italian writers associated with Paul of Venice (d. 1429), several of whose disciples wrote commentaries on the portions of Heytesbury's *Regule* concerned with local motion. These are preserved in a Venice edition of 1494,[17] which served to keep the "calculatory" tradition alive on the Continent long after it had ceased to be of interest in Great Britain. The most interesting of these treatises is the commentary of Gaetano da Thiene (d. 1465), who showed a knowledge of the terminology of Schemata I and II, but who proposed yet another alternative for classifying the various types of local motion.[18] This, like Schema I, influenced many authors and in fact dominated the tradition through the first half of the sixteenth century, up to the time of Soto's writing.

Gaetano differentiates uniform from difform motion both with respect to time and with respect to the parts of the object moved, as had earlier authors. He departs from them, however, in introducing a sixfold grouping that makes use of the two-variable concept of Albert and Oresme but allows for two more possibilities. For Gaetano a motion may be uniform either with respect to the parts of the object moved *alone*, or with respect to time *alone*, or with respect to the parts and to time *taken together*.[19] Similarly, a motion may be difform in the same three ways. The resulting six possibilities, written in an improvised notation (in which $\sim$ stands for negation), are given in Schema III.

[16] *Mechanics in the Middle Ages,* pp. 553–555; see also Anneliese Maier, *An der Grenze von Scholastik und Naturwissenschaft,* 2. Auflage (Rome: Edizioni di Storia e Letteratura, 1952) pp. 214–215, 315–316.

[17] *Hentisberi de sensu composito et diviso* . . . (Venice: Bonetus Locatellus, 1494). This edition is described by Curtis Wilson, *William Heytesbury: Medieval Logic and the Rise of Mathematical Physics* (Madison: Univ. Wisconsin Press, 1960), p. 4, n., and *passim.*

[18] On Gaetano see P. Silvestro da Valsanzibio, O.F.M. Cap., *Vita e dottrina di Gaetano di Thiene, filosofo dello studio di Padova, 1387–1465* (2nd ed., Padua: Studio Filosofico dei Fratrum Minorum Cappuccini, 1949).

[19] *Expositio litteralis supra tractatum [Hentisberi] de tribus [predicamentis], De motu locali* (Venice: 1494), fol. 37^{rb}:

> Unde notandum quod motus uniformis potest esse triplex: primo quo ad subiectum tantum; secundo quo ad tempus tantum et non quo ad subiectum; tertio quo ad subiectum et tempus simul.

SCHEMA III

$$
v \left\{ \begin{array}{ll}
U \left\{ \begin{array}{lll} U(x) & \cdot \sim U(t) & \ldots\ldots\ldots \text{ III-1} \\ U(t) & \cdot \sim U(x) & \ldots\ldots\ldots \text{ III-2} \\ U(x) & \cdot\, U(t) & \ldots\ldots\ldots \text{ III-3} \end{array} \right. \\
D \left\{ \begin{array}{lll} D(x) & \cdot \sim D(t) & \ldots\ldots\ldots \text{ III-4} \\ D(t) & \cdot \sim D(x) & \ldots\ldots\ldots \text{ III-5} \\ D(x) & \cdot\, D(t) & \ldots\ldots\ldots \text{ III-6} \end{array} \right.
\end{array} \right.
$$

This schema is redundant: III-2 is equivalent to III-4, and III-1 is equivalent to III-5. Such redundancy, however, was quite common in medieval systems of division.

Having enumerated these possibilities, Gaetano gives an example of each one, thereby setting a precedent for most of those who were to adopt this particular method of classification. He illustrates type III-1 with the heavy object (*grave*) falling and type III-2 with the wheel (*rota*), as had Albert of Saxony for the analogous cases in his schema (II-2 and II-3); the same two illustrations but in the reverse order he attaches to types III-4 (*rota*) and III-5 (*grave*). His example for type III-3, on the other hand, is ambiguous. Gaetano speaks of a body descending "in a uniform space" (*mobile descendit in spacio uniformi*) and regards this as a motion that is uniform with respect both to time and to the parts of the falling object.[20] He makes no mention of the resistance increasing with the interval of fall, as does Albert of Saxony for the analogous case (II-1), and this seems in fact to be ruled out by his expression *in spacio uniformi*.[21] An alternative possibility could be an object being lowered at constant speed—not falling freely—but there is no positive suggestion of this in the text. Gaetano's example for the remaining case, type III-6, on the other hand is new—a wheel (*rota*) whose angular velocity is being continually increased (*movetur velocius et velocius*). Finally he mentions the example of a ball (*pila*) that falls and rotates as it does so: various components of its motion then illustrate the different types. Falling, it exemplifies type III-1; if rotating uniformly it exemplifies type III-2; if turning slower and slower, it exemplifies type III-6.[22]

There are other examples in Gaetano's commentary that are of interest. He mentions an object moving rectilinearly and supposes that it is neither contracting

[20] *Ibid.*:
Exemplum tertii, ut quando mobile descendit in spacio uniformi, et non velocitatur magis in una parte temporis quam in alia: tunc movetur equaliter et uniformiter quo ad magnitudinem, scilicet, quo ad omnia sua puncta, quorum tantum precise descendit unum sicut reliquum, et in equali parte temporis.

[21] The words *in spacio uniformi* are themselves puzzling, since 15th-century writers had no conception to match the modern notion of isotropic space. The words probably refer to a uniform *medium* and not to a uniform *space* understood in the strict sense.

[22] *Ibid.*:
Unde notandum quod quando pilla descendit rotando tunc ille motus multipliciter consideratur; nam inquantum descendit sic talis motus est uniformis quo ad subiectum et difformis quo ad tempus; quando vero pilla rotat, tunc aut rotat equevelociter aut tardius et tardius—si equevelociter ille motus est uniformis quo ad tempus et difformis quo ad subiectum; et postea considero illos duos motus, si secundo modo est difformis quo ad subiectum et quo ad tempus.

nor expanding as it moves, for the expansion or contraction would obviously cause a nonuniform motion of some of its parts. Along the same line he proposes the case of a wheel that rotates, but he now imagines the wheel to expand and contract during its rotation—a phenomenon that would explain further difformities in the motion of its parts. Another example is the placing of a cutting edge against a wheel to continually cut off the outermost surface, thus producing a difformity of the motion of the circumference. A more imaginative possibility is to have the inner parts of the wheel expand while the outermost surface is being cut off; this produces a more complicated variation in the difformity of the movement of the parts. Yet another example is a disc made of ice that is rotated in a hot oven: here the outermost surface continually disappears, and the velocity at the circumference becomes slower and slower; whereas the inner parts expand under the influence of the heat, and their linear velocity increases. A final example is that of a wheel that rotates and continually has material added to its circumference, as a potter might add clay to the piece he is working. Although the velocity of rotation is uniform, the linear velocity of a point on the circumference would increase, unless the entire wheel could be made to contract in the process, in which case it would remain constant.[23]

The foregoing examples can all be viewed as variations of the types of motion sketched in Schema III. Gaetano mentions also some of the types of motion that occur in Schema I; he gives definitions or kinematical descriptions of uniformly difform (I-5 and I-6) and difformly difform (I-7 and I-8) motions, but in these cases he follows the English *Calculatores* and gives no examples whatsoever—not even those already provided by John of Holland.

The remaining Italian commentaries on Heytesbury's *Regule* show more affinity with the latter part of Gaetano's commentary than with its earlier sections in which examples abound: the commentators restrict themselves, for the most part, to kinematical descriptions. Thus Messinus divides local motion into uniform and difform motion and gives a definition of uniform motion that applies only to uniformity with respect to time; he does stipulate, however, that the moving object must retain its quantitative dimensions throughout the motion, thereby implying that there be no change of the parts with respect to each other, and he further stipulates that the "space" passed over be neither contracting nor expanding during the motion.[24] When speaking of difform local motion he makes explicit the distinction between difformity with respect to time and difformity with respect to magnitude and says that an infinite number of possibilities exist for both types of difformity. The relation between distance and time can have any ratio one might wish, and the variation of velocities between respective parts of the moving object can be anything imaginable.[25]

[23] Gaetano, *Expositio litteralis*, fols. 37ra–40rb. These examples may seem overly fanciful to the modern reader, and yet without their help it becomes almost impossible to visualize the complex kinematical cases being discussed by Gaetano and his contemporaries.

[24] *Questio Messini de motu locali* (Venice: 1494), fol. 52va:

> Motus uniformis est per quem continue manentem equalem nec remissum nec intensum tantum de spacio pertransitur in uno tempore sicut in alio sibi equali, ceteris paribus, puta quod spacium non maneat condempsatum nec rarefactum, et mobile sit semper eiusdem quantitatis.

To speak of *spacium* as *condempsatum* or *rarefactum* is as puzzling as to refer to it as *uniforme* (see n. 21 above).

[25] *Ibid.*, fol. 52va:

> Motus difformis in infinitum variari potest: tam respectu temporis quam respectu magnitudinis; potest enim plus de spacio pertransiri in uno tempore quam in alio, scilicet, equali; et hoc in quacumque proportione volueris. Similiter in diversis temporibus inequalibus in quacumque proportione volue-

Angelo da Fossombrone,[26] on the other hand, reflects some of the concern for exemplification that is found in Gaetano da Thiene. Angelo's characterization of local motion, for example, stresses its priority in the order of nature and its essential division into upward and downward, the distinctions of uniform and difform being considered accidental. Like Messinus he is concerned with eliminating physical factors that would cause the moving body to expand or contract or would change the dimensions of the space through which it moves, and he wishes to define the types of local motion so as to rule out such possibilities.

Angelo follows Gaetano's Schema III, moreover, but supplies only three examples: for type III-3 he gives a moving object (*mobile*) that does not change its parts through condensation or rarefaction but continues in rectilinear motion with unchanging velocity; for type III-2 he cites a wheel (*rota*) that revolves and is moved by a constant force (*potentia*) exerted upon it; and for type III-1 in place of the customary falling body he provides the more abstract example of an object (*mobile*) all of whose parts move with the same velocity as the whole while this velocity is changing with respect to time.[27]

Bernardo Torni of Florence,[28] alone of this group, reverts to Schema II and mentions three of its four possibilities—the identical ones discussed by Albert of Saxony. He also mentions a few cases that would fit into Schema I. However, he gives no examples—intentionally, since he believes the reader knows enough to furnish his own (*casus tuipse scis formare*).[29]

From this survey of fifteenth-century Italy we can see that the development

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ris potest spacium pertransiri equale; et in infinitum variari potest ex parte temporis; similiter ex parte magnitudinis eo quod una pars potest moveri velocius alia certa data. Unde talis motus est difformis: quo ad partes subiecti. Et hoc est motum variari respectu magnitudinis; et illud infinitis modis potest esse, sicut satis patet: quia qualitercumque velis imaginari quod una pars, alia velocius moveatur.

[26] *Supra tractatu[m] de motu locali* (Venice: 1494), fols. 64^{ra}–73^{rb}.

[27] *Ibid.*, fol. 64^{ra}:

Motuum autem localium tam uniformium quam difformium aliquis est talis, scilicet, uniformis vel difformis quo ad tempus; aliquis quo ad partes subiecti mobilis. Et aliquis potest esse motus uniformis quo ad tempus et etiam quo ad partes subiecti: ut si unum mobile moveatur non variatum intrinsecus per condensationem aut rarefactionem semper eodem gradu motus non intenso nec remisso motu simpliciter recto movetur uniformiter quo ad tempus, ut apparet per dictam descriptionem. Movetur etiam uniformiter quo ad partem subiecti moti, quia omnes partes omnino velocitate consimili moventur. Moveri enim uniformiter quo ad partes subiecti est omnes partes eque velociter moveri. Aliquid enim contingit uniformiter moveri quo ad tempus et difformiter quo ad partes subiecti, sicut rota mota circulariter ab eadem potentia continue equali nixu ita quod circumferentia ipsius continue equaliter movetur. Tunc patet quod totum movetur difformiter quo ad partes subiecti, quia velocius moventur partes versus circumferentiam quam versus centrum. Aliquid tamen movetur uniformiter quo ad partes subiecti et difformiter quo ad tempus sicut si tali motu simpliciter recto aliquod mobile moveatur omnino semper equali velocitate quo ad omnes suas partes qua movetur totum cum hoc tamen in partibus temporis intendendo vel remittendo motum suum, etc.

[28] *In capitulum de motu locali Hentisberi quedam annotata* (Venice: 1494), fols. 73^{va}–77^{va}.

[29] *Ibid.*, fols. 74^{rb}–74^{va}:

Scias tamen quod communiter ponitur ista distinctio. Aliquid uniformiter moveri intelligitur dupliciter: quo ad tempus, et quo ad partes subiecti; et eodem modo dupliciter contingit difformiter moveri. In superibus locuti sumus quo ad tempus. Verum id uniformiter movetur quo ad partes quando quelibet eque velociter cum alia movetur; difformiter vero quando una velocius et alia tardius. Ex quo patet quod stat *A* difformiter moveri quo ad tempus et uniformiter quo ad partes subiecti, et e contra. Et stat quo ad utrumque uniformiter moveri; et casus tuipse scis formare.

The last phrase suggests that examples were either discussed in class or left to the students as exercises.

there was somewhat ambivalent. As evidenced in the work of Gaetano, there is a concern for exemplification, with insistence on the case of the falling body, but this appears in the setting of Schema III with its two-variable classification similar to that used at the University of Paris, where *uniformiter difformis* with respect to time does not appear. As evidenced in the remaining commentators on Heytesbury, on the other hand, there are occasional references to types of motion that fit into Schemata I and II (including *uniformiter difformis*), but for these there is only kinematical description, no exemplification.

Sixteenth-Century Paris

A pronounced revival of interest in physical problems took place at Paris in the early part of the sixteenth century under the influence of the Scottish nominalist Jean Mair, whose disciples wrote a considerable number of "questionaries" on the *Physics* of Aristotle.[30] In these it was customary to incorporate treatments of local motion that borrowed heavily from such writers as Heytesbury, and thus there was once again a fusion of Mertonian and Parisian thought. Again, in this period a considerable number of scholars from the Iberian peninsula were studying at Paris, and as a result there was a diffusion of the new developments into Spain and Portugal within a few decades of their discussion at Paris. The various schemata we have been discussing figure in this new movement, and there is a growth of exemplification that prepares for the association of uniformly difform motion with the case of falling bodies.

The first writer to prepare for this association was the Augustinian John Dullaert of Ghent, who edited the works of Paul of Venice, another Augustinian, and also wrote questions on the *Physics*.[31] Dullaert was a disciple of Jean Mair, and he seems to have brought about a blending of Parisian nominalist interests deriving from Albert of Saxony with the realist concerns that characterized the school of Paul of Venice.

Dullaert's exposition follows Schema III and gives illustrations for all six possibilities.[32] For type III-1 there is the usual example of the heavy body (*grave*) falling through a uniform medium; this case illustrates type III-5 also. For types III-2 and III-4 Dullaert uses Oresme's example of the motions of the heavenly sphere (*sphera celestis*) rather than the customary wheel. The illustration of type III-3 is Albert of Saxony's: a body falling through space whose resistance is so proportioned that it has uniform velocity with respect both to time and to all the parts of the subject. Type III-6, finally, is exemplified by the wheel (*rota*) that accelerates its rotation—identical with the case provided by Gaetano da Thiene.[33]

[30] On Jean Mair (Johannes Maior) and his school see Hubert Élie, "Quelques Maitres de l'université de Paris vers l'an 1500," *Archives d'histoire doctrinale et littéraire du Moyen Age,* 1950–1951, *18*:193–243; also R. G. Villoslada, S.J., *La Universidad de Paris durante los estudios de Francisco de Vitoria, O.P. (1507–1522)* (Rome: Gregorian Univ. Press, 1938), pp. 127–278.

[31] *Questiones super octo libros phisicorum Aristotelis necnon super libros de celo et mundo* (Lyons: 1512); Élie cites two previous editions, Paris 1506 and Paris 1511.

[32] *Ibid., Phisica,* lib. 3, quest. 1, fol. 65va:
Quarto notandum est quod duplex est motus localis: quidam est uniformis et quidam difformis. Et triplex est motus uniformis: quidam est uniformis quo ad subiectum tantum, quidam quo ad tempus, quidam quo ad tempus et subiectum simul. . . . Similiter triplex est motus difformis, scilicet, quo ad subiectum, quo ad tempus, et quo ad utrumque.

[33] *Ibid.*:
Motus alicuius gravis cuius motus est velocior in fine quam in principio est uniformis sed non regularis, et motus alicuius sphere celestis est regularis et nullo modo uniformis, sed sto cum communi modo loquendi nec secum volo contendere de nomine.

Dullaert, however, does not stop here; he mentions some of the categories of Schema I and significantly furnishes a few examples also. For type I-3 he gives the motion of a heavenly sphere (*sphera celestis*) the parts of which move with uniformly increasing velocity as one goes from the pole to the equator. For type I-5 he cites the case of Socrates (*Sortes*) uniformily increasing his walking speed from zero to eight degrees—the example of John of Holland—and for type I-6 he mentions the converse case of Socrates decelerating his motion uniformly from a given speed to zero. He then explains that types I-7 and I-8 would be defined "in the opposite manner" (*opposito modo*), without giving examples.[34]

Apart from these divisions Dullaert mentions twice the velocity of descent of a falling body. In the first instance he gives it as an illustration of Albert of Saxony's terminology, as being a uniform but not a regular motion. The second mention comes when Dullaert refers to his exemplification of falling motion as being faster at the end than at the beginning, saying that some wonder how this can occur, since it would seem that the same ratio of force over resistance is maintained and thus the motion should be uniform. Dullaert postpones discussion of this case but says that the motion is actually faster at the end than at the beginning because of the accidental impetus that is built up in the fall.[35] He does not state that the motion will be uniformly difform, but is content to illustrate uniformly difform motion in terms of Socrates' walking speed.

Writing shortly after Dullaert, the Portuguese Alvaro Thomaz prepared a lengthy treatise *De triplici motu* patterned on the work of Swineshead but also incorporating materials from Gaetano da Thiene and others.[36] Like the English Mertonians, Alvaro is more concerned with kinematical descriptions of various types of motions than he is with examples drawn from the physical universe. He follows the initial classifications of Schema I, dividing motion into uniform and difform, and difform in turn into uniformly difform and difformly difform. Without defining these he immediately subdivides uniformly difform into a threefold classification similar to that used by Gaetano da Thiene: with respect to the subject *alone*, which may be symbolized $UD(x) \cdot \sim UD(t)$; with respect to time *alone*, symbolized $UD(t) \cdot \sim UD(x)$; and with respect to both subject and time *together*, symbolized $UD(x) \cdot UD(t)$.[37] He then gives the same threefold classification for

[34] *Ibid.*:
Et iterum duplex est motus difformis: quidam uniformiter difformis, quidam difformiter difformis. Et aliquis est motus uniformiter difformis quo ad subiectum, et est quandocumque partis subiecti dimidium tantum exceditur in velocitate ab extremo velocius moto quantum excedit aliud extremum, ut motus sphere celestis. De motu uniformiter difformi quo ad tempus ex illo facile patet quid sit dicendum, ut si Sortes deberet intendere motum suum per unam totam horam a non gradu usque ad 8, tunc quacumque parte illius motus accepta medium tantum excedit extremum quantum exceditur ab altero extremo. Et eodem modo si Sortes remitteret motum suum uniformiter ab aliquo certo gradu usque ad non gradum. Et opposito modo debet diffiniri motus difformiter difformis.

[35] *Ibid.*, fol. 65^{vb}:
Sed quia tangitur ibi de corpore gravi quod velocius descendit in fine quam in principio, dubitaret aliquis, et merito, unde hoc perveniat. Et supponamus quod si aliquod corpus grave ut 8 quid debeat moveri deorsum per aliquod medium subduple resistentie, tunc ex quo manebit semper eadem proportio activitatis super resistentiam sequitur quod non velocius descendit in fine quam in principio.—Sed ad hoc dicitur quod semper manebit eadem proportio essentialis sed non accidentalis, videlicet propter impetum. . . . Alias de hoc futurus est sermo.

[36] *Liber de triplici motu proportionibus annexis . . . philosophicas Suiseth calculationes ex parte declarans* (Paris: 1509).

[37] *Ibid.*, pars 3, tract. 2, cap. 1 (no foliation):
Motus uniformiter difformis (ut communiter definitur) est triplex: quidam est

difformly difform motion and mentions that it can also be applied to uniform motion. The implied schema, a variant of Schema III, is sufficiently different from this to be included in our study as something new, which we shall designate as Schema IV.

SCHEMA IV

$$v \begin{cases} U \begin{cases} U(x) \cdot \sim U(t) & \ldots\ldots \text{IV-1} \\ U(t) \cdot \sim U(x) & \ldots\ldots \text{IV-2} \\ U(x) \cdot U(t) & \ldots\ldots \text{IV-3} \end{cases} \\ D \begin{cases} UD \begin{cases} UD(x) \cdot \sim UD(t) & \ldots\ldots \text{IV-4} \\ UD(t) \cdot \sim UD(x) & \ldots\ldots \text{IV-5} \\ UD(x) \cdot UD(t) & \ldots\ldots \text{IV-6} \end{cases} \\ DD \begin{cases} DD(x) \cdot \sim DD(t) & \ldots\ldots \text{IV-7} \\ DD(t) \cdot \sim DD(x) & \ldots\ldots \text{IV-8} \\ DD(x) \cdot DD(t) & \ldots\ldots \text{IV-9} \end{cases} \end{cases} \end{cases}$$

Of the nine possibilities contained here, Alvaro discusses only three in detail, types IV-4 through IV-6, and he gives only one example, for type IV-4, the motion of the potter's wheel (*rota figuli*), although he does give a kinematical description of type IV-5 (which approximates I-5), an object (*aliquod mobile*) that moves from zero to a given velocity, uniformly increasing its speed.[38]

Another student of Jean Mair at Paris was the Spaniard Luis Coronel, a townsman of Domingo de Soto (both were from Segovia), who was possibly teaching at Paris when Soto came there as a young student. In his *Physice perscrutationes*[39] Coronel discusses many topics that were commonly dealt with in "questionaries" on the *Physics,* but he is sparing in his treatment of the types of local motion. He mentions, in passing, the division of local motion into rectilinear and curvilinear, and in discussing how the relative motions of two objects are to be compared, he states that either a uniformly difform or a difformly difform velocity would have to be reduced to an average value before a comparison could be effected. He gives no definitions of these types of motions, however, and provides no examples. Rather he refers the reader to the treatises of Heytesbury and Swineshead, with which he seems generally to agree, and otherwise does not think it worthwhile to waste his time over such matters.[40]

A similar treatment is to be found in the *Physica* of Juan de Celaya, another Spaniard who taught at Paris and who definitely numbered among his students Domingo (then Francisco) de Soto.[41] Celaya is not only acquainted with the

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uniformiter difformis quo ad subiectum tantum, quidam quo ad tempus tantum, quidam vero quo ad subiectum et tempus simul.

[38] *Ibid.*:

. . . quo ad subiectum . . . Exemplum ut motus rote figuli . . . quo ad tempus . . . Exemplum ut si aliquod mobile incipiat moveri a non gradu continuo intendendo uniformiter motum suum per aliquod tempus; tunc talis motus est uniformiter difformis quo ad tempus.

[39] *Physice perscrutationes* (Paris: 1511); for a summary of Coronel's teaching see Pierre Duhem, *Études sur Léonard de Vinci,* Vol. III (Paris: Hermann, 1913), pp. 552–555.

[40] Coronel, *Physice,* lib. 3, pars 3, fol. 86^{rb}.

[41] See Beltrán de Heredia, *Domingo de Soto,* p. 16.

writings of Heytesbury and Swineshead, but makes explicit mention of the Italian commentators on Heytesbury. What is of particular interest in Celaya's exposition, however, is his departure from the two-variable type of schema (i.e., Schemata II, III, and IV) that dominated these treatments from Albert of Saxony to Alvaro Thomaz, and his return to the one-variable classification of Schema I. Although, like Coronel, he gives no examples, Celaya defines six of the eight types of motion in Schema I.[42] Possibly he was the writer who influenced Soto to adopt this classificatory schema in preference to the others, for he is the first among Soto's immediate predecessors, to my knowledge, to make use of it; besides, as already noted, he did teach Soto. Yet strangely enough neither he nor any other writer at Paris in his time seems to have thought to associate motion that is uniformly difform with respect to time with the case of falling bodies.

Sixteenth-Century Spain

The Spaniards we have discussed thus far studied at Paris and wrote their treatises while there. Other Spaniards under Parisian influence wrote questions on the *Physics* of Aristotle at universities in Spain: principal among these are Diego Diest,[43] whose *Questiones phisicales* appeared at the University of Saragossa in 1511; Diego de Astudillo,[44] who wrote at Valladolid in 1532; and Domingo de Soto, whose physical works were first published at Salamanco *circa* 1545. Since nominalist treatises seem to have received a less enthusiastic reception in Spain than they had in Paris, these writers, all of whom undertook to incorporate "calculatory" concepts into their courses on Aristotle, were careful to show that such concepts relate in some way to the physical universe. Partially for this reason and partially for pedagogical reasons they utilized considerably more exemplification than did those who wrote in Paris. This, coupled with the diversity of schemata that now seemed to require exemplification, set the stage for a new look at the old examples, for the introduction of some new ones, and for the eventual association of motion that is *uniformiter difformis* with the case of falling bodies.

Diest treats of uniform and difform motions at length, first in the context of Schema III, and then in a less systematic way that mentions elements of Schemata I and IV while discussing uniformly difform motion. He treats all six possibilities in Schema III, giving an example of each: for types III-1 and III-5 he cites the motion of a heavy object (*grave*) downward;[45] for types III-2 and III-4 he gives

[42] *Expositio . . . in octo libros phisicorum Aristotelis, cum questionibus . . . secundum triplicem viam beati Thome, realium, et nominalium* (Paris: 1517), fols. 81^{rb}–83^{vb}. Although giving no examples, Celaya does refer the reader to his treatment of uniformly difform qualities for a further understanding of these definitions: "Et qualiter iste diffinitiones habent intelligi ex declaratione qualitatis uniformiter difformis inferius apparebit" (fol. 81va).

[43] Diest, a native of Bolea in Spain, studied at Paris in the latter part of the 15th century. The full title of his work is *Questiones phisicales super Aristotelis textum, sigillatim omnes materias tangentes in quibus difficultates que in theologia et aliis scientiis ex phisica pendent discusse suis locis inseruntur* (Saragossa: 1511). For some further details on Diest, see Villoslada, *Universidad de Paris*, pp. 401–402.

[44] Astudillo taught at the College of Saint Gregory in Valladolid, which was staffed by Dominicans; here he composed his *Questiones super octo libros phisicorum et super duos libros de generatione Aristotelis* (Valladolid: 1532).

[45] Diest, *Questiones*, lib. 3, quest. 3, fol. 16va:

> Motus uniformis quoad subiectum tantum est motus uniformis quo aliquod mobile et quaelibet eius pars uniformiter movetur et difformiter quoad tempus, ut motus gravis deorsum totum et quaelibet eius pars uniformiter movetur sed difformiter quoad tempus quia velocius movetur in fine quam in principio.

the example of the wheel (*rota*);[46] for type III-3 he mentions a heavy ball (*sperula gravis*) falling downward in a medium that continually offers more and more resistance so that the velocity remains uniform;[47] and for type III-6 he suggests a wheel (*rota*) rotating with varying angular velocity.[48]

Having given and exemplified Gaetano's division of difform motion, Diest returns to this subject and provides an alternative division of difform motion into uniformly difform and difformly difform. He then embarks on a discussion of *uniformiter difformis* that is extremely interesting, for in it he states that the *uniformiter* part of this expression may be understood in various ways, meaning by this uniform variation either in a linear sense or in a logarithmic sense, which is clearly an innovation when compared to previous applications of this expression to the velocity of falling motion.

Diest first explains "uniformly" in the linear sense, as it had been commonly understood by his predecessors. He then proceeds to a second way of defining uniformly difform motion as follows:

> [This type of] uniformly difform motion occurs when a change in intensification, velocity, or quantity corresponds immediately to an extensive change in proportionable parts; briefly, when there is the same excess of the first proportionable part over the second as the second over the third, and so on.[49]

This statement is cryptic, and is explained in what follows immediately:

> This appears in local motion: it is commonly taught that a heavy object falling downward increases its speed uniformly difformly, so that it moves with a greater velocity in the second proportionable part than in the first, and with greater velocity in the third than in the second, and so on.[50]

The expression "proportionable part" is used interchangeably by Diest for "proportional part," and it means the same as geometric (or, in modern terminology, logarithmic) part. Thus Diest is saying that the velocity increase is the same in the first half of the body's fall as it is in the next quarter, as in the next eighth, and so on. In other words, the velocity of a falling object increases geometrically with the

46 *Ibid.*:
Motus uniformis quoad tempus tantum est motus quo aliquod mobile et qualibet eius pars non uniformiter movetur, in equalibus tamen partibus temporis equalia spacia pertransit tale mobile, ut in rota mota notum est quod non quaelibet pars rote cum qualibet alia uniformiter movetur quia quanto aliqua pars magis accedit ad centrum tanto minus movetur . . . et tamen uniformiter movetur quo ad tempus, quia rota tantum spacium pertransit in una parte temporis quantum in alia sibi equali.

47 *Ibid.*:
Motus dicitur uniformis quoad utrumque quando totum et quelibet pars uniformiter movetur et in equalibus partibus temporis equalia spacia pertranseunt, sicut est in linea mota uniformiter motu recto: potest exemplificari in sperula gravi mota deorsum supposito quod in medio continue maiorem et maiorem resistentiam inveniat taliter quod propter illam uniformiter moveatur in principio et in fine quo ad tempus.

48 *Ibid.*:
Patet exemplum in rota mota magis in uno tempore quam in alio sibi equali.

49 *Ibid.*:
Alio modo diffinitur sic: motus uniformiter difformis est quando in tali partes proportionabiles immediate secundum extensionem sunt immediate secundum intensionem vel velocitatem vel quantitatem; breviter quod qualis est excessus prime partis proportionabilis ad secundam talis est secunde ad tertiam, et sic procedendo.

50 *Ibid.*:
Patet in motu locali: dicitur communiter quod grave descendendo doersum movetur uniformiter difformiter taliter intendendo motum quod velocius movetur in secunda parte proportionabili quam in prima, et in tertia magis quam in secunda, et sic procedendo.

distance of fall, going to infinity over a finite range. This, it must be noted, Diest proposes as common teaching (*dicitur communiter*), a statement that offers difficulty when one considers that Albert of Saxony had already considered this possibility only to reject it and that there seems to have been little or no discussion of it by the intervening authors. A way out of the difficulty would be to read into Diest's statement the understanding that the velocity increases by proportional parts, corresponding to the proportional parts of the distance traversed, which would be equivalent to holding that it increases linearly with distance of fall. This seems to have been "common teaching" from Albert of Saxony onward, and may have been what Diest intended, although the textual exegesis does not favor this interpretation.

This example, again, might appear to be an illustration of motion that is of type I-5, that is, uniformly difform with respect to time; actually it is not, for the independent variable in Diest's presentation is spatial (*partes secundum extensionem*) and not temporal—and here he foreshadows a difficulty that was to plague Galileo (in his early writings) and others in their attempts to formulate a correct law of falling bodies.[51]

Diego de Astudillo, like Soto, was a Dominican, and he was a close friend of one of Soto's first Dominican professors, the eminent jurist Francisco de Vitoria.[52] Astudillo's questions on the *Physics* (1532) cite Diest and most of the authors we have already mentioned, although his treatment is briefer than Diest's and not of as great significance. He works for the most part within Gaetano's schema, defining and exemplifying all six of its types. For type III-1 he provides the example of "all natural movements, for a stone [*lapis*] falling downward moves with equal velocity in all its parts, although with respect to time the velocity is greater toward the end than at the beginning, as is obvious from experience."[53] The reference to "experience" is significant, although it clearly cannot be taken in any metrical sense. The same example, as was usual, he associates with type III-5. For types III-2 and III-4 he prefers the illustration of the heavens (*celum*), as did Dullaert, giving as his (erroneous) reason that "the parts closer to the poles traverse more space than do those that are more remote."[54] For type III-3 he gives Albert of Saxony's example, observing that "this motion only seems capable of occurring per accidens, by reason of a resistance variation; e.g., if a stone [*lapis*] falls downward and encounters increasing resistance in the same proportion as its velocity of descent would be naturally increased."[55] The latter statement might be taken to imply that

[51] One of the pioneer discussions of this topic is Alexandre Koyré, *Études galiléennes*, Fasc. II: *La loi de la chute de corps—Descartes et Galilée* (Paris: Hermann, 1939).

[52] Both Astudillo and Vitoria taught at Saint Gregory's in Valladolid. Of Astudillo, Vitoria graciously remarked that he knew far more than himself but was not as good at marketing his ideas: "Fray Diego de Astudillo más sabe que yo, pero no vende tan bien sus cosas." See Villoslada, *Universidad de Paris*, pp. 304–305.

[53] *Op. cit.*, lib. 6, quest. 4, fol. 117^{rb}:

Motus uniformis quoad subiectum est cuius omnes partes subiecti equa velocitate moventur ex parte subiecti, licet non ex parte temporis: sed maiori ex parte temporis et minori. Exemplum patet in omnibus motibus naturalibus. Lapis enim descendens deorsum, quantum ad omnes partes eque velociter movetur, tamen ex parte temporis, maior est velocitas circa finem quam principium, ut ex experientia patet.

[54] *Ibid.*:

. . . ut celum: quelibet enim pars celi tantum spacium describit in una hora quantum in alia, tamen non omnes partes equalia describunt spacia. Maiora enim spacia describunt partes pole propinquiores quam remotiores [sic].

[55] *Ibid.*:

Iste motus solum videtur inveniri posse per accidens ratione variationis alicuius resistentiae. Ut si lapis descendens deorsum, secundum proportionem secundum quam descendens naturaliter augeretur in velocithe stone's natural fall is uniformly accelerated, but at best this is only an inference; there is no clear indication, moreover, how "uniformly" should then be taken, particularly considering Diest's difficulties with the alternative meanings of this term. Finally, for type III-6 Astudillo introduces a new example, that of a "violent circular motion [*motus circularis violentus*] resulting from a projecting,"[56] evidently meaning by this some type of impelled rotation that comes gradually to rest, in which case there would be a velocity variation with respect both to parts and to time.

Astudillo, like Diest, gives the further twofold division of difform motion into uniformly difform and difformly difform and divides the first into two types, "with respect to the subject" and "with respect to time," thus touching on the various members of Schema I, types I-3 through I-8. He illustrates type I-3 with the heavenly body (*corpus celeste*), but merely defines the other types. After his definition of type I-5, that is, uniformly difform with respect to time, he adds cryptically, "as is apparent from the above."[57] This could mean that Astudillo thought he had already discussed this case in terms of an example or that the definition was so clear in light of the foregoing examples that it needed no illustration. Which meaning one takes depends on how he evaluates Astudillo's examples of the falling stone already discussed. I favor the latter alternative.

This brings us finally to Domingo de Soto, who had read Astudillo and most of the other authors already discussed, although he generally refrained from mentioning them by name. What is most remarkable about Soto is that he breaks completely with his immediate predecessors in rejecting all the two-variable schemata (II, III, and IV) and returns instead to Heytesbury's one-variable schema (I), which had been used by Juan de Celaya alone of all of the sixteenth-century writers. Soto gives a full explanation of this schema and then, in the fashion that had by then become customary, supplies examples for all its types. It is in this setting that he finally associates falling bodies with *uniformiter difformis* motion, taking *uniformiter difformis* in the precise sense of motion uniformly accelerated in time, to which he can, and does, apply the Mertonian "mean-speed theorem."

Soto gives the complete division of Schema I, plus definitions of all its types. Here we can only enumerate his examples, most of which had already been used by one or more of his predecessors.[58] For type I-1 he gives the case of a foot-length of stone being drawn over a plane surface (*si . . . pedalem lapidem trahas super planitiem*), while for type I-2 he mentions the invariant motion of the heavens (*in regulatissimo motu celorum perspectum est*), and for type I-3, the rotation of a millstone (*mola frumentaria*). Type I-4 offers more difficulty: Soto is unable to supply an example that involves local motion and so gives one that involves changes of quality (alteration), and this for the case of heating. His example is "a four-foot long object that is so altered in one hour that its first foot uniformly takes on a degree of heat of one, and its second uniformly the degree of two, and its third the

tate inveniant [sic] augmentum resistentie. Tunc enim utroque modo esset motus uniformis.

56 *Ibid.*, fol. 117va:

. . . sicut patet in motu circulari violento, qui est per proiectionem.

57 *Ibid.*:

Et patet ex dictis.

58 The following citations are from Soto's *Super octo libros physicorum questiones* (Salamanca: 1555), lib. 7, quest. 3; the earliest complete edition of this work was published at Salamanca in 1551 although an earlier printing, lacking parts of Bk. 7 and all of Bk. 8, appeared there *c.* 1545. There are no known manuscripts of the text. I plan to publish the principal portions of the Latin text with all significant variants together with an English translation and a commentary.

degree of three, etc."—for which he supplies the diagram of a step-function. He goes on to observe that "the present treatise is not at all concerned with this kind of alterative motion" but that he has given this example for the simple reason that a local motion of this type is hardly possible: rectilinear motion must be uniform in this respect, and rotary motion can only be uniformly difform—by its nature it cannot be difformly difform.[59]

Types I-5 and I-6 Soto defines in conjunction with each other and then observes that they are "properly found in objects that move naturally and in projectiles." He goes on:

> For when a heavy object falls through a homogeneous medium from a height, it moves with greater velocity at the end than at the beginning. The velocity of projectiles, on the other hand, is less at the end than at the beginning. And what is more, the [motion of the] first increases uniformly difformly, whereas the [motion of the] second decreases uniformly difformly.[60]

Then later on in the text while discussing the same case, "uniformly difform motion with respect to time," he removes any possible ambiguity as to his meaning by proposing the difficulty "whether the velocity of an object that is moved uniformly difformly is to be judged from its maximum speed, as when a heavy object falls in one hour with a velocity increase from 0 to 8, should it be said to move with a velocity of 8?"[61] His answer to this is clearly in terms of the Mertonian "mean-speed theorem," for he decides in favor of the average velocity (*gradus medius*) as opposed to the maximum. He justifies this with the illustration: "For example, if the moving object *A* keeps increasing its velocity from 0 to 8, it covers just as much space as [another object] *B* moving with a uniform velocity of 4 in the same [period of] time."[62] Thus there can be no doubt about his understanding of *uniformiter difformis* and how this is to be applied to the space traversed by a freely falling object.

To exemplify types I-7 and I-8, finally, Soto resorts to the motion of animals and to other biological changes, stating:

> An example would be if something were to move for an hour, and for some part [of the hour] were to move uniformly with a velocity of one, and for another [part] with a velocity of two, or three, etc., as is experienced in the progressive motion of animals. This kind of motion frequently occurs in the alteration of animals' bodies, and perhaps it can take place in the motion of augmentation and diminution.[63]

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[59] Soto, *ed. cit.*, fol. 92^{vb}:
Et licet de hac specie motus alterationis nihil ad presens negotium, subiecimus tamen hoc exemplum idcirco, quod motus localis haud quaquam esse potest difformiter difformis quo ad subiectum. Quoniam rectus quidem nequit ullo modo difformis: omnes enim partes continue equaliter moventur; circularis vero, omnis est uniformiter difformis.
Obviously Soto is thinking only of the rotation of a rigid, nondeformable body and not the more imaginative cases mentioned by Gaetano da Thiene (see above).

[60] *Ibid.*:
Hec motus species proprie accidit naturaliter motis et proiectis. Ubi enim moles ab alto cadit per medium uniforme, velocius movetur in fine quam in principio. Proiectorum vero motus remissior est in fine quam in principio; atque adeo primus uniformiter difformiter intenditur, secundus vero uniformiter difformiter remittitur.

[61] *Ibid.*, fol. 93^{vb}:
Utrum velocitas mobilis uniformiter difformiter moti sit denominanda a gradu velocissimo, ut si grave decidat in una hora velocitate a non gradu usque ad 8, dicendus sit moveri ut 8?

[62] *Ibid.*, fol. 94ra:
Exempli gratia, si *A* mobile una hora moveatur intendendo semper motum a non gradu usque ad 8 tantumdem spatii transmittet quantum *B*, quod per simile spatium eodem tempore uniformiter moveretur ut 4.

[63] *Ibid.*, fol. 92^{vb}:
. . . ut si ita res aliqua moveretur per horam, ut per aliquam partem uniformiter

The illustrations apply mostly to type I-7, but his mention of diminution at the end (although not strictly a local motion) indicates that he is also aware of decreasing variations and thus implicitly includes type I-8 in his exemplification.

The further details of Soto's analysis of falling motion, together with its influence on later thinkers, must await treatment elsewhere. The materials presented here, however, should help clear up at least part of "the enigma of Domingo de Soto." The contribution of the Spanish Dominican was not epoch-making, but it was significant nonetheless. Of the nineteen authors considered in this paper he alone thought of systematically providing examples for the simplest of the four schemata used—that which considers only one independent variable at a time. The others who were interested in exemplification—and these were mostly late-fifteenth-century or sixteenth-century writers—worked in the context of two-variable schemata, and this generally precluded the possibility of their even considering the case of motions that are uniformly difform with respect to time.[64] All of Soto's examples, of course, like those of his predecessors, were proposed as intuitive, without empirical proof of any kind. Moreover, he and Diest, of all those considered, were the most venturesome in attempting to assign a precise quantitative modality to falling motion. Of the two, Soto was without doubt the better simplifier; he seems also to have been the better teacher, and he was philosophically more interested in unifying the abstract formulations of the nominalists with the physical concerns of the realists of his day.[65] Again, he had the advantage of time and of being able to consider more proposals. The strange alchemy of the mind that produces scientific discoveries requires such materials on which to work. It goes without saying that Soto could not know all that was implied in the simplification he had the fortune to make. But then, neither could Galileo, in his more refined simplification, as the subsequent development of the science of mechanics has so abundantly proved.

moveretur ut 1, et per aliam ut 2 vel 3, etc. Ut est experiri in motibus progressivis animalium. Que quidem species motus crebro accidit in alteratione corporum animalium, et potest forsan contingere in motu augmenti et decrementi.

64 The exception is Alvaro Thomaz, who did mention motion that is uniformly difform with respect to time in the context of his two-variable schema. His division, however, was so complex as to discourage any attempts at simple exemplification with natural examples.

65 For a further discussion of this last point see my paper "The Concept of Motion in the Sixteenth Century," *Proceedings of the American Catholic Philosophical Association* (Washington, D.C.: Catholic University of America, 1967) pp. 184–195.